

Tips for the first round of the Georg Mohr Contest

Algebra

Here we present techniques to solve algebra problems. It is not a theoretical exposition, but instead focused on exemplifying some central principles and ideas about how to, for example, gauge the size of a number, convert a problem into a formula or an equation, solving equations and systems of equations, and manipulating inequalities. To succeed in the first round of the Georg Mohr Contest, the most important thing is to practice a lot of exercises. Therefore, this document contains a collection of problems, many of them previously used as first round Georg Mohr questions.

In the first round of the Georg Mohr Contest, the first ten problems are multiple choice questions with five answers to choose from, while the last ten problems have answers in the form of positive integers. The problems here are a mix of these two kinds. There are solution sketches for all problems in the back.

Determining the size of a number

To determine which of two numbers is greater, you often need to rewrite the numbers so they are in a comparable form.

It is usually hard to judge the size of a square root, so if one of the two numbers contain a square root and you want to determine which is greater, you can often compare the squares of the numbers to get rid of the square root.

Example To determine which of the following numbers is the greatest,

$$\text{A) } \tfrac{4}{3} \quad \text{B) } \sqrt{2} \quad \text{C) } \tfrac{16}{13} \quad \text{D) } \tfrac{\sqrt{10}}{3}$$

we first compare the two fractions A and C:

$$\frac{4}{3} > \frac{16}{13}, \quad \text{since} \quad 4\cdot 13 > 3\cdot 16.$$

C cannot be the answer.

To compare A, B and D we square the numbers, as two of them contain square roots:

$$\text{A) } \left(\frac{4}{3}\right)^2 = \frac{16}{9}, \quad \text{B) } (\sqrt{2})^2 = 2, \quad \text{D) } \left(\frac{\sqrt{10}}{3}\right)^2 = \frac{10}{9}.$$

Because B has the greatest square, it must be the greatest of the original numbers, as they are all positive.

Problem 1. Which of the following numbers is greatest?

$$\text{A) } \tfrac{1}{4} \quad \text{B) } 0{,}47 \quad \text{C) } \tfrac{4}{7} \quad \text{D) } (\tfrac{1}{4})^2 \quad \text{E) } \tfrac{7}{40}$$

(*Georg Mohr 2010, problem 1*)

Problem 2. Which of the following numbers is smallest?

$$\text{A) } \tfrac{3}{103} \quad \text{B) } \tfrac{1}{33} \quad \text{C) } \tfrac{301}{1001} \quad \text{D) } \tfrac{3}{100} \quad \text{E) } \tfrac{30}{1003}$$

(*Georg Mohr 2014, problem 6*)

Problem 3. Which of the following numbers is greatest?

$$\text{A) } \sqrt{408} \quad \text{B) } 20 \quad \text{C) } \sqrt{100}+\sqrt{308} \quad \text{D) } \tfrac{\sqrt{816}}{2} \quad \text{E) } 2\sqrt{204}$$

(*Georg Mohr 2012, problem 15*)

Problem 4. Which of the following numbers is greatest?

$$\text{A) } 2^3\cdot 3^4\cdot 4^3 \quad \text{B) } (4\cdot 3^2)^2\cdot(3\cdot 4^2)\cdot 2 \quad \text{C) } 2\cdot 3\cdot \tfrac{4}{2\cdot 3}\cdot 3\cdot \tfrac{3^3}{2^4}\cdot 2^{10}$$
$$\text{D) } 2^{10}\cdot 3^4 \quad \text{E) } \tfrac{3^4}{2}\cdot 4^5$$

(*Georg Mohr 2007, problem 15*)



Finding an expression

In the next set of problems, we want to find an expression given some information. We are given some variables, and from these variables we want to find an expression for something that depends on these variables.

Example It takes 5 minutes to fill out a specific form on a computer. There are x people who have to fill out the form over the span of an hour. We want to find an expression for how many computers are needed.

To find an expression for this, we first note that there are 60 minutes in an hour, and therefore $\frac{60}{5} = 12$ people per computer who can fill out the form over the span of an hour. If x people must fill out the form over the span of an hour, $\frac{x}{12}$ computers are required. This number must of course be rounded up to an integer, as we only work with whole computers.

Problem 5. An ice cream cone with two scoops is 15 kroner, a cone with three scoops is 20 kroner, a cone with four scoops is 25 kroner, and so on. How much is a cone with n scoops of ice cream?

A) $15n+5$ B) $5n+10$ C) $5n+5$ D) $n+15$ E) $(n-2)\cdot 5+10$

Problem 6. At a party, n nice 8-person tables of the type shown are placed end to end. How many seats does one get?



A) n B) $7n+1$ C) $n(n-2)+2$ D) $6n+2$ E) $8(n-1)$

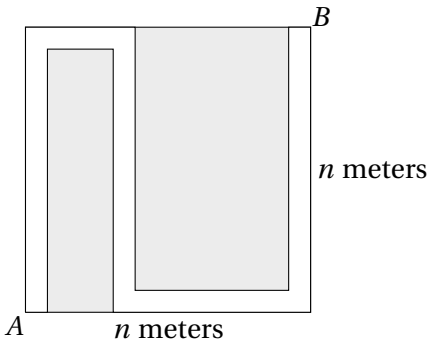
(*Georg Mohr 2009, problem 2*)

Problem 7. For a party, each of n persons prepares s liters of soup. The soup is to be filled into big jars holding d liters each. At least how many jars are needed when each jar must only be half filled?

A) $\frac{2ns}{d}$ B) $\frac{\frac{1}{2}ds}{n}$ C) $\frac{2s}{dn}$ D) $\frac{nds}{2}$ E) $\frac{ns}{2d}$

(*Georg Mohr 2010, problem 9*)

Problem 8. In an exhibition area measuring n meters by n meters, the audience is led through the exhibition along a 1 meter wide path from the corner A to the corner B as shown in the figure.



How many m^2 is left for the exhibition when the area of the path is deducted?

A) n^2-3n+1 B) n^2-4n+3 C) n^2-n-5 D) n^2-4n E) $n^2-3n-10$

(*Georg Mohr 2017, problem 2*)

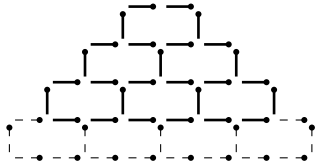
Problem 9. At a gathering of p people there are s liters of juice per person. The juice is poured into cans each containing m liters, and the cans are distributed on n tables. How many cans are there on each table on average?

A) $\frac{s\cdot p}{m\cdot n}$ B) $\frac{p\cdot m}{n\cdot s}$ C) $\frac{m\cdot s}{p\cdot n}$ D) $\frac{p\cdot n}{s\cdot m}$ E) $\frac{n\cdot m}{p\cdot s}$

(*Georg Mohr 2018, problem 6*)



Problem 10. Adam is building a figure with many rows. He just finished row no. n. How many matches does he need to add to the figure so that it has $n+1$ rows?



A) $n+1$ B) $n+(n+1)+2+(n+2)$ C) $3(n+2)$
D) $2n+2(n+1)$ E) $2n$

(*Georg Mohr 2011, problem 12*)

Equations

When you've got an equation, you can check if a specific number is a solution, or you can solve the equation.

If you have more than one equation, you can combine them and investigate what you can, in total, deduce from the equations.

Sometimes you have to introduce your own variables fitting the problem, and set up a system of equations that you then solve.

Example To check if something is a solution to an equation, you plug it into the equation.

For example, $x=\frac{1}{2}$ is not a solution of $\frac{x}{4}+\frac{8}{x}=3$, as

$$\frac{\frac{1}{2}}{4}+\frac{8}{\frac{1}{2}}=\frac{1}{8}+16\neq 3.$$

However, $x=4$ is a solution to the equation $\frac{x}{4}+\frac{8}{x}=3$, as

$$\frac{4}{4}+\frac{8}{4}=3.$$

Problem 11. In which of the following equations is $x=5$ a solution?

A) $(x-1)+(x-6)=0$ B) $(x-4)^2+(x-5)^2=0$
C) $(x-3)^3+(x-7)^3=0$ D) $(x-5)^4+(x-2)^4=0$
E) $(x-1)^5+(x-5)^5=0$

(*Georg Mohr 2010, problem 6*)

Problem 12. The equation

$$\frac{x}{3}+\frac{5}{x}=45x+x^2$$

has three solutions. One of the solutions is:

A) $x=3$ B) $x=\frac{1}{3}$ C) $x=5$ D) $x=\frac{1}{5}$ E) $x=\frac{1}{15}$

(*Georg Mohr 2011, problem 13*)

Problem 13. If $\frac{1}{n+3}=\frac{1}{7}$, then $\frac{1}{n^2+9}$ is equal to

A) $\frac{2}{7}$ B) $\frac{1}{14}$ C) $\frac{1}{49}$ D) $\frac{1}{16}$ E) $\frac{1}{25}$

(*Georg Mohr 2010, problem 3*)

Example To solve the equation

$$(x+2)^2+(2x+4)^2=0,$$

we use that the square of a number cannot be negative. Because the sum of $(x+2)^2$ og $(2x+4)^2$ has to be zero, each term must be zero. This means that both $x+2=0$ and $2x+4=0$, which is true only when $x=-2$. Therefore, $x=-2$ is the only solution to the equation.

Problem 14. How many real numbers x satisfy

$$(x-3)^2+(2x-8)^2+(2x-3)^4+(x-4)^4=0\,?$$

A) 0 B) 1 C) 2 D) 3 E) 4

(*Georg Mohr 2009 s, problem 11*)



Example Now, we'll look at what you can conclude from some equations. The numbers a, b og c satisfy that $ab+bc=2$ og $a^2+b^2+c^2=3$. From this, what can you deduce with certainty?

A) a, b and c are all positive
B) a, b and c are all negative
C) if a and c are negative, then b is negative
D) at least one of the three numbers is 0

As $(a,bc)=(1,1,1)$ and $(a,b,c)=(-1,-1,-1)$ are both solutions to the system of equations, you cannot deduce either A, B or D. If a and c are both negative, it means that $a+c$ is negative. As $(a+c)b=ab+bc=2$, b must also be negative. Therefore, we can deduce C.

Problem 15. The three integers a, b and c satisfy $a+b-1=b+c=c+a+3$. Which of the numbers a, b and c is greatest?

A) a B) b C) c D) they are all equal E) it cannot be decided

(*Georg Mohr 2015, problem 4*)

Problem 16. t is known that the number $x=4$ is a solution to this equation: $a+b\cdot\sqrt{x^3+36}=c\cdot x$.
What can be deduced concerning the equation $a+b\cdot\sqrt{x^6+36}=c\cdot x^2$?

A) $x=2$ is a solution B) $x=4$ is a solution C) $x=8$ is a solution
D) $x=16$ is a solution E) none of these

(*Georg Mohr 2016, problem 4*)

Problem 17. The number a is known to satisfy

$$2017a^{2017}+100=110.$$

What is

$$2017(-a)^{2017}+100\,?$$

(*Georg Mohr 2017, problem 13*)

Problem 18. Three numbers x, y and z satisfy $xy+z=0$. What can be deduced from this?

A) all the numbers x, y and z are 0
B) $x^2y^2+z^2=0$
C) at least one of the numbers x, y og z is negative
D) if the number z is 0, at least one of the numbers x and y is 0, as well
E) if the number x is negative, at least one of the numbers y and z is positive

(*Georg Mohr 2011, problem 8*)

Problem 19. The numbers x, y, z and w satisfy $x+2y=3z+w$ and $z+2w=3x+y$. Which of the following statements is not necessarily correct?

A) $y+w=2(z+x)$ B) $x-3z=w-2y$ C) $4x+3y=4z+3w$
D) $z+2w=3(3z+w-2y)+y$ E) $z+3x=y+2w$

(*Georg Mohr 2012, problem 19*)

Problem 20. The numbers a, b, c and d satisfy the equations:

$$a+b-c=d,\quad a-b+c=d,\quad -a+b-c=d,\quad -a-b+c=d.$$

What can be deduced from this?

A) $a=b$ and $d=0$ B) $b=c$ and $a=0$ C) $a=c$ and $d=0$
D) $b=d$ and $a=0$ E) $a=d$ and $c=0$

(*Georg Mohr 2006, problem 11*)

Problem 21. Which positive integer n satisfies

$$n(n+3)(n+6)\cdots(n+297)(n+300)=(200-n)(203-n)(206-n)\cdots(497-n)(500-n)\,?$$

(*Georg Mohr 2017, problem 15*)



Example - Introduce variables and find equations
Emma collects ice-cream sticks, beer-bottle caps and stickers. When calculating how much her collection is worth, she usually assumes that four ice-cream sticks equal seven beer-bottle caps and that ten stickers equal three beer-bottle caps. How many stickers equal six ice-cream sticks?

(*Georg Mohr 2012, problem 3*)

To answer this, we first introduce variables: Denote the value of a ice-cream stick by i, the value of a beer-bottle cap o, and the value of a sticker k. We know that $4i = 7o$ and $10k = 3o$, so $i = \frac{7}{4}o$ and $o = \frac{10}{3}k$. Therefore,

$$6i = 6 \cdot \frac{7}{4}o = \frac{21}{2}o = \frac{21}{2} \cdot \frac{10}{3}k = \frac{21 \cdot 10}{2 \cdot 3}k = 35k.$$

So six ice-cream sticks are worth 35 stickers.
But you can also be clever and directly see that 12 ice-cream sticks are worth 21 beer-bottle caps that are worth 70 stickers, and therefore, 6 ice-cream sticks are worth 35 stickers.

Problem 22. A number of bicycles, tricycles and four-wheeled strollers are parked in the court- yard. There are twice as many strollers as tricycles, and there are three times as many bicycles as strollers. Altogether, there are 184 wheels. How many tricycles are there?

(*Georg Mohr 2013, problem 11*)

Problem 23. Anna needs to finish two subjects in order to have completed her edu- cation. If she gets 12 in both of them the average of her marks will be exactly 8. If she gets 2 in both subjects it will be exactly 7. Of how many subjects does the education consist?

(*Georg Mohr 2011, problem 15*)

Problem 24. The sum of 81 consecutive integers is 3^8. What is the middle of the 81 numbers?

(*Georg Mohr 2017, problem 17*)

Problem 25. n an election in two rounds with the candidates P1, P2 and P3, only P1 and P2 go on to the second round. In the first round, P3 got 25 % of the votes. His voters are expected to distribute themselves in the second round with 20 % to P1 and 80 % to P2, who is then expected to obtain a total of 55 % of the votes. How big was P1?s part of the votes in the first round?

(*Georg Mohr 2008, problem 11*)

Inequalities

We'll look at how to manipulate inequalities and determine how big a variable is, or how big it is compared to another. This section is markedly more difficult than the rest.

Working with inequalities

You can add or subtract a number on both sides of the inequality symbol:

If $a < b$, then $a \pm c < b \pm c$.

You can multiply by a positive number on both sides of the inequality symbol:

If $a < b$ and $c > 0$, then $a \cdot c < b \cdot c$.

You can divide both sides of the inequality symbol by a positive number:

If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

You can add two inequalities:

If $a < b$ and $c < d$, then $a + c < b + d$.

You can square both sides if both sides are positive:

If $a < b$ and a and b are positive numbers, then $a^2 < b^2$.

You can take the square root of both sides if both sides are positive: If $a < b$ and a and b are positive numbers, then $\sqrt{a} < \sqrt{b}$.



Example The numbers a and b are both between 0 and 1, that is, $0 < a < 1$ and $0 < b < 1$. Which of the following numbers does not necessarily share this property?

A) $\frac{a+b}{2}$ B) $\sqrt{ab}$ C) $\sqrt{\frac{a}{2}}+\sqrt{\frac{b}{2}}$ D) $\frac{ab}{a+b}$

(*Georg Mohr 2009, problem 17*)

A) First, we add the inequalities $0 < a < 1$ and $0 < b < 1$ and get $0 < a+b < 2$. Then we divide by 2 and get $0 < \frac{a+b}{2} < 1$. The number $\frac{a+b}{2}$ is therefore also between 0 and 1.

B) When the inequality $0 < a < 1$ is multiplied by b, we get $0 < ab < b$. This is allowed because b is positive. Because $0 < ab$, $0 < \sqrt{ab}$. Now, we use the fact that $b < 1$, implying $ab < b < 1$. Because $ab < 1$ where both a and b are positive, we can square root both sides, giving us $\sqrt{ab} < \sqrt{1} = 1$. The number $\sqrt{ab}$ is therefore also between 0 and 1.

D) As in B, $0 < ab < b$. We divide by the positive number $a+b$ and get $\frac{ab}{a+b} < \frac{b}{a+b}$. The fraction $\frac{b}{a+b}$ must be smaller than one, as the denominator is larger than the numerator. Therefore, $\frac{ab}{a+b} < 1$. As both numerator and denominator of $\frac{ab}{a+b}$ are positive, it must be greater than 0. The number $\frac{ab}{a+b}$ is therefore also between 0 and 1.

C) The number $\sqrt{\frac{a}{2}}+\sqrt{\frac{b}{2}}$, on the other had, is not always between 0 and 1. If fx. $a = b = \frac{1}{2}$, then $\sqrt{\frac{a}{2}}+\sqrt{\frac{b}{2}} = \sqrt{\frac{1}{4}}+\sqrt{\frac{1}{4}} = \frac{1}{2}+\frac{1}{2} = 1$.

Problem 26. What can you deduce from knowing that $a < b+c$ and $b < a+c$?

A) $c = 0$ B) $a = b$ C) $a+b < a+b+c$ D) $c < 0$ E) $\frac{a+b}{2} < c$

Problem 27. Three positive numbers x, y and z are known to satisfy $2x > 3y > 4z$. Which of these can be deduced?

A) $x < y < z$ B) $x > 2y > 3z$ C) $3x > 4y > 5z$
D) $3x > 5y > 7z$ E) $x > 3y > 5z$

(*Georg Mohr 2017, problem 4*)

Problem 28. Let a be a number satisfying $0 < a < 1$. Which of the following statements is true?

A) $a > \sqrt{a}$ B) $\frac{1}{a} > a$ C) $a > \frac{1}{\sqrt{a}}$
D) $a^3 > 1$ E) $a^2 > a$

Problem 29. You are given that the two numbers x and y satisfy

$$4x \leq 3y + 2016 \leq 2x + 2016.$$

What is the largest possible value of y?

(*Georg Mohr 2016, problem 18*)

Problem 30. Determine the least integer n such that

$$x^{10} - 1001x^7 + 1 > 0$$

for all $x \geq n$.

(*Georg Mohr 2014, problem 19*)

Problem 31. A positive integer n has the property that there exists exactly one positive integer k such that

$$54n < 55k < 56n \ .$$

What is the greatest possible value of n?

(*Georg Mohr 2013, problem 20*)



Assorted problems

Problem 32. What is the number of solutions to the equation

$$-x(x-1)+x(x-2)-x(x-1)(x-2)=0\,?$$

(*Georg Mohr 2007, problem 11*)

Problem 33. A positive integer is called a *palindrome* if it is identical to its mirror image. For example, 57475 is a palindrome, while 1227 is not. You are given that x is a four-digit palindrome, and that $x+852$ is a 5-digit palindrome. What is x?

(*Georg Mohr 2016, problem 15*)

Problem 34. Set $a_0=3$ and $a_1=5$ and, thereafter,

$$a_2=a_1-a_0, \quad a_3=a_2-a_1, \quad \ldots$$

What happens to the numbers a_n in the long run?

A) the numbers get greater and greater B) from a certain step all the numbers are positive

C) from a certain step all the numbers are negative D) the numbers approach 0

E) the numbers repeat themselves periodically

(*Georg Mohr 2010, problem 19*)

Problem 35. Determine the number

$$\left(1+\frac{1}{2}\right)^2\left(1+\frac{1}{3}\right)^2\left(1+\frac{1}{4}\right)^2\left(1+\frac{1}{5}\right)^2\ldots\left(1+\frac{1}{999}\right)^2.$$

(*Georg Mohr 2018, problem 19*)



Solution sketches

Problem 1 Answer: C). Because $\frac{4}{7} > \frac{1}{2}$, while all the other numbers are smaller than $\frac{1}{2}$, $\frac{4}{7}$ is the biggest number.

Problem 2 Answer: A). It's true that $A < E < D < B$ and $A < C$ because

$$\frac{30}{1030} < \frac{30}{1003} < \frac{30}{1000} < \frac{30}{990} \quad \text{og} \quad \frac{30}{1030} < \frac{301}{1001}.$$

Remember that when the numerator increases, the fraction increases, and when the denominator increases, the fraction decreases.

Problem 3 Answer: E). First, B, D and E are rewritten as square roots:

$$\text{B)}\ 20 = \sqrt{400}$$

$$\text{D)}\ \frac{\sqrt{816}}{2} = \frac{\sqrt{816}}{\sqrt{4}} = \sqrt{\frac{816}{4}} = \sqrt{204}$$

$$\text{E)}\ 2\sqrt{204} = \sqrt{4}\sqrt{204} = \sqrt{4 \cdot 204} = \sqrt{816}.$$

Now, A) $\sqrt{408}$, B) $\sqrt{400}$, D) $\sqrt{204}$ and E) $\sqrt{816}$, and it's clear that E is the greatest of these.

Now, we consider which of E and C is greatest:

$$\text{C)}\ \sqrt{100} + \sqrt{308} = 10 + \sqrt{308} < 10 + 18 = 28, \text{ as } 308 < 18^2 = 324$$

$$\text{E)}\ \sqrt{816} > 28, \text{ as } 816 > 28^2 = 784.$$

Therefore, E is greatest.

Problem 4 Answer: B). We rewrite all the expressions to the form $2^n \cdot 3^m$, to more easily compare their sizes:

A) $2^2 \cdot 3^4 \cdot 4^3 = 2^3 \cdot 3^4 \cdot (2^2)^3 = 2^3 \cdot 3^4 \cdot 2^6 = 2^9 \cdot 3^4$.

B) $(4 \cdot 3^2)^2 \cdot (3 \cdot 4^2) \cdot 2 = (2^2 \cdot 3^2)^2 \cdot 3 \cdot (2^2)^2 \cdot 2 = 2^4 \cdot 3^4 \cdot 3 \cdot 2^4 \cdot 2 = 2^9 \cdot 3^5$.

C) $2 \cdot 3 \cdot \frac{4}{2 \cdot 3} \cdot 3 \cdot \frac{3^3}{2^4} \cdot 2^{10} = \frac{2 \cdot 3 \cdot 2^2 \cdot 3 \cdot 3^3 \cdot 2^{10}}{2 \cdot 3 \cdot 2^4} = \frac{2^{13} \cdot 3^5}{2^5 \cdot 3} = 2^8 \cdot 3^4$.

D) $2^{10} \cdot 3^4$.

E) $\frac{3^4}{2} \cdot 4^5 = \frac{3^4 \cdot (2^2)^5}{2} = \frac{3^3 \cdot 2^10}{2} = 2^9 \cdot 3^3$.

It's clear that A, C and E are smaller than B. As $2^{10} \cdot 3^4 = 2^9 \cdot 3^4 \cdot 2 < 2^9 \cdot 3^4 \cdot 3 = 2^9 \cdot 3^5$, B is greatest.

Problem 5 Answer: C). Note that the price increases by 5 kroner for each extra scoop. Therefore, it costs 5 kroner per scoop. Furthermore, the cone is 5 kroner, as a cone with two scoops is 15 kroner, one with with scoops is 20 kr, and so on. A cone with n scoops therefore costs $5n+5$ kroner.

Problem 6 Answer: D). Along the sides of each table there's room for 6 people, meaning there's room for $6n$ people along the sides of n tables. Furthermore, there's a place at each end of the long table, that is, 2 extra people. When n tables are combined, we therefore have room for $6n+2$ people.

Problem 7 Answer: A). Because n people each make s liters of soup, there is a total of ns liters of soup. Each jar can contain $\frac{d}{2}$ liters of soup. Therefore, we need at least $\frac{ns}{\frac{d}{2}} = \frac{2ns}{d}$ jars.

Problem 8 Answer: B). The three vertical paths are n meters long and 1 meter wide, so they have area $3n$ m^2. The two horizontal paths are, combined, $n+1$ meter long and 1 meter wide, so they have area $n+1$ m^2. There are 4 path components, each 1×1 meter, that are part of both a horizontal and a vertical path, so we've counted those twice. Therefore, the total path area is $3n + (n+1) - 4 = 4n - 3$. That makes the exhibition area $n^2 - (4n-3) = n^2 - 4n + 3$.

Problem 9 Answer: A).

Because there are p people, and we need s liters of juice per person, we need $p \cdot s$ liters of juice. Because each can can contain m liters, we need $\frac{ps}{m}$ cans. Because there are n tables, we on average need $\frac{ps}{nm}$ cans per table.

Problem 10 Answer: C). To expand the figure to $n+1$ rows, we need two more matches for the upper edge of row $n+1$. Additionally, we need $n+2$ matchsticks for the vertical part, and $2(n+1)$ matchsticks for the horizontal part. In total, we need

$$2 + (n+2) + 2(n+1) = 3(n+2).$$

Problem 11 Answer: C). By inserting $x = 5$ in the five equations, we can see that $x = 5$ is only a solution of C.



A) $(5-1)+(5-6)=3\neq 0$

B) $(5-4)^2+(5-5)^2=1\neq 0$

C) $(5-3)^3+(5-7)^3=2^3+(-2)^3=0$

D) $(5-5)^4+(5-2)^4=3^4\neq 0$

E) $(5-1)^5+(5-5)^5=4^5\neq 0$

Problem 12 Answer: B).By inserting the five options into the equation, we can see that only $x=\frac{1}{3}$ is a solution. Here, we only show the calculation for $x=\frac{1}{3}$. We first compute the left side, then the right side:

$$\frac{x}{3}+\frac{5}{x}=\frac{\frac{1}{3}}{3}+\frac{5}{\frac{1}{3}}=\frac{1}{9}+15 \quad \text{and} \quad 45x+x^2=45\cdot\frac{1}{3}+\left(\frac{1}{3}\right)^2=15+\frac{1}{9}.$$

Problem 13 Answer: E). First we solve the equation $\frac{1}{n+3}=\frac{1}{7}$:

$$\frac{1}{n+3}=\frac{1}{7}\Leftrightarrow n+3=7\Leftrightarrow n=4.$$

So $\frac{1}{n^2+9}=\frac{1}{4^2+9}=\frac{1}{25}$.

Problem 14 Answer: A). First, note that none of the four terms in

$$(x-3)^2+(2x-8)^2+(2x-3)^4+(x-4)^4$$

can be negative. Because their sum has to equal 0, they must all be equal to 0. This means that $x-3=0$, $2x-8=0$, $2x-3=0$ and $x-4=0$ simultaneously, but there is no x with this property. Therefore, the equation has no solutions.

Problem 15 Answer: B). We have that $a+b-1=c+a+3$ and therefore $b=c+4$, that is, b is greater than c. Furthermore, we have that $b+c=c+a+3$ and therefore $b=a+3$, that is, b greater than a. So b is greatest..

Problem 16 Answer: A). Because $x=4$ is a solution of $a+b\cdot\sqrt{x^3+36}=c\cdot x$, we can see that

$$a+b\cdot\sqrt{4^3+36}=c\cdot 4 \quad\Leftrightarrow\quad a+b\cdot\sqrt{2^6+36}=c\cdot 2^2.$$

This shows that $x=2$ is a solution of $a+b\cdot\sqrt{x^6+36}=c\cdot x^2$.

Problem 17 Answer: 90. Because $2017a^{2017}+100=110$, we know that $2017a^{2017}=10$ implying that $2017(-a)^{2017}=-2017a^{2017}=-10$. Now, $2017(-a)^{2017}+100=-10+100=90$.

Problem 18 Answer: D). If $z=0$, then $xy=0$. By the zero-product property, that means that at least one of the numbers is 0. This allows us to deduce D. Because $(x,y,z)=(0,0,0)$ and $(x,y,z)=(-1,-1,-1)$ are both solutions of the equation, we can exclude A, B, C and E being true.

Problem 19 Answer: E).

A) $y+w=2(z+x)$ is obtained by adding the two equations.

B) $x-3z=w-2y$ is obtained by rewriting $x+2y=3z+w$.

C) $4x+3y=4z+3w$ is obtained by adding the two equations: left side in the first equation added to right side on the second, and vice versa.

D) $z+2w=3(3z+w-2y)+y$ is obtained by solving for x in $x+2y=3z+w$ and substituting it into $z+2w=3x+y$.

E) $z+3x=y+2w$ is not always correct, as for example the solution $(x,y,z,w)=(-1,3,2,-1)$ doesn't satisfy this condition.

Problem 20 Answer: B. The first two equations gives us that $a+b-c=d=a-b+c$ and therefore $2b=2c\Leftrightarrow b=c$. If that is substituted into the first equation, we get that $a=d$. If we substitute it into the last equation, we get that $-a=d$. This means that $-a=a$, that is, $a=0$. This lets us deduce B. We cannot deduce A, C, D og E, as $a=d=0$ and $b=c=1$ are solutions to all the equations.

Problem 21 Answer: 100. On the left side, we have

$$n(n+3)(n+6)\cdots(n+297)(n+300)$$

which is a product of 101 numbers, the smallest of which is n, and the next number is 3 larger than the previous. On the right side, we have

$$(200-n)(203-n)(206-n)\cdots(497-n)(500-n)$$

which is the product of 101 numbers, the smallest of which is $200-n$, and the next number is 3 larger than the previous. This means that $n=200-n$ for equality to hold, meaning $n=100$.



Problem 22 Answer: 8. Denote the number of tricycles x. This means we have $2x$ strollers and $3 \cdot 2x = 6x$ bicycles. In total, we then have

$$184 = 3 \cdot x + 4 \cdot 2x + 2 \cdot 6x = 23x$$

wheels. This tells us that $x = \frac{184}{23} = 8$, so we have 8 tricycles.

Problem 23 Answer: 20. Denote the number of subjects n. As her average is exactly 8 if she gets a 12 in the two last subjects, the sum of her grades in the subjects she's already finished must be

$$8n - 2 \cdot 12 = 8n - 24.$$

As her average is exactly 7 if she gets a 02 in the two last subjects, the sum of her grades in the subjects she's already finished can also be written as

$$7n - 2 \cdot 2 = 7n - 4.$$

So

$$8n - 24 = 7n - 4,$$

implying $n = 20$. The education therefore consists of 20 subjects.

Problem 24 Answer: 81. Denote the middle number in the list of the 81 numbers n. There are 40 numbers before and after n. As the sum of the 81 numbers is 3^8, we know that

$$3^8 = (n-40)+(n-39)+\cdots+(n-1)+n+(n+1)+\cdots+(n+39)+(n+40) = 81n = 3^4 n.$$

This means that $n = \frac{3^8}{3^4} = 3^4 = 81$.

Problem 25 Answer: 40. Let $x\%$ be the share of votes that P2 got in the first round. In the first round, P3 got 25% of the votes, and because his voters are expected to split in the second round with 20% going to P1 and 80% going to P2, we expect P2 to get $x\% + 0{,}8 \cdot 25\% = x\% + 20\%$ in the second round. As we also know that P2 is expected to get a total of 55% of the votes in the second round, we have the equation:

$$x\% + 20\% = 55\%.$$

So $x = 55 - 20 = 35$. This means that in the first round, P1 got

$$100\% - 25\% - 35\% = 40\%$$

of the vote.

Problem 26 Answer: C). By adding the inequalities, we get

$$a + b < a + b + 2c.$$

This means that $0 < 2c$, implying $0 < c$. Therefore, we can deduce that C)

$$a + b < a + b + c.$$

Because $(a, b, c) = (10, 11, 2)$ satisfies both inequalities, we cannot deduce any of A, B, D and E.

Problem 27 Answer: C). First, note that $x = 5$, $y = 3$ and $z = 2$ satisfies the inequality. By substituting $x = 5$ and $y = 3$ into the inequalities A, B, D and E, we see that none of them are satisfied, so we cannot deduce any of those statements.

If $2x > 3y$ then $3x = \frac{3}{2} \cdot 2x > \frac{3}{2} \cdot 3y = \frac{9}{2} y > 4y$, so we can deduce that $3x > 4y$.

If $3y > 4z$ then $4y = \frac{4}{3} \cdot 3y > \frac{4}{3} \cdot 4z = \frac{16}{3} z > 5z$, so we can deduce that $4y > 5z$. This allows us to deduce C.

Problem 28 Answer: B). If $0 < a < 1$, then $a^2 < 1$. Because a is positive, we can divide both sides by a and get $a < \frac{1}{a}$. Therefore, B) is true. The other statements are not true for any a i intervallet, for example, not for $a = \frac{1}{4}$:

A) $a > \sqrt{a}$ is not true because $\frac{1}{4} < \sqrt{\frac{1}{4}} = \frac{1}{2}$.

C) $a > \frac{1}{\sqrt{a}}$ is not true because $\frac{1}{4} < \frac{1}{\sqrt{\frac{1}{4}}} = 2$.

D) $a^3 > 1$ is not true because $\left(\frac{1}{4}\right)^3 = \frac{1}{64} < 1$.

E) $a^2 > a$ is not true because $\left(\frac{1}{4}\right)^2 = \frac{1}{16} < \frac{1}{4}$.

Every statement but B) can actually be rewritten as $a^x > 1$ for som positive number x, and that is not true when $0 < a < 1$.



Problem 29 Answer: 672. The inequalities show that $4x \leq 2x + 2016$ so $2x \leq 2016$. The last inequality shows that $3y \leq 2x$, and combined with $2x \leq 2016$ we get $3y \leq 2016$, that is, $y \leq \frac{2016}{3} = 672$. This is possible for $y = 672$ when $x = \frac{2016}{2} = 1008$. So 672 is the greatest possible value of y.

Problem 30 Answer: 11. First, rewrite the inequality:

$$x^7(x^3 - 1001) + 1 > 0.$$

Now, we observe that the left side is negative if $1 \leq x \leq 10$, and positive if $x \geq 11$, because $10^3 < 1001 < 11^3$. Therefore $n = 11$ is the least integer such that the inequality is satisfied for all $x \geq n$.

Problem 31 Answer: 55. First, rewrite the inequality by dividing by 55:

$$\frac{54}{55}n < k < \frac{56}{55}n.$$

If $n = 55$, the inequality becomes $54 < k < 56$, and there exists exactly one positive integer k satisfying that. If $n \geq 56$, then

$$\frac{54}{55}n < n < \frac{56}{55}n,$$

and

$$\frac{54}{55}n < n + 1 < n + \frac{1}{55}n = \frac{56}{55}n.$$

In that case, both $k = n$ and $k = n + 1$ satisfy the inequality, so there exist at least two positive integers satisfying the inequality.

Problem 32 Answer: 1. First, x is factored out of the expression:

$$\begin{aligned}
0 &= -x(x-1) + x(x-2) - x(x-1)(x-2) \\
0 &= x(-(x-1) + (x-2) - (x-1)(x-2)) \\
0 &= x(-x + 1 + x - 2 - (x^2 - 3x + 2)) \\
0 &= x(-x^2 + 3x - 3).
\end{aligned}$$

By the zero-product property, $x = 0$ or $-x^2 + 3x - 3 = 0$. The discriminant of this quadratic is negative, so it has no solutions. This means that $x = 0$ is the only solution, so the equation has exactly 1 solution.

Problem 33 Answer: 9449. First, notice that x has four digits and $x + 852$ has five digits. This means that $x \geq 10000 - 852 = 9048$. Because x is a palindrome, $x = 9aa9$, where a is a digit. Furthermore, $9aa9 + 852 \leq 9999 + 852 = 10851$. This means that the five-digit palindrome $9aa9 + 852$ is of the form $10b01$, where b is a digit. This means that $a9 + 52 = 101$, implying that $a9 = 101 - 52 = 49$ and $a = 4$. Therefore, $x = 9449$ and $x + 852 = 10301$.

Problem 34 Answer: E. If you calculate the first few numbers in the sequence, we get: $3, 5, 2, -3, -5, -2, 3, 5, \ldots$. As the next number in the sequence is determined by the two previous ones, we can see that the sequence repeats and becomes periodic when we get back to the two first numbers in the same order. This means the sequence consists of the numbers $3, 5, 2, -3, -5, -2$ repeated periodically forever. This means we can exclusive A, B, C and D, and that E is true.

Problem 35 Answer: 250000. We rewrite the product P:

$$\begin{aligned}
P &= \left(1 + \frac{1}{2}\right)^2 \left(1 + \frac{1}{3}\right)^2 \left(1 + \frac{1}{4}\right)^2 \left(1 + \frac{1}{5}\right)^2 \ldots \left(1 + \frac{1}{999}\right)^2 \\
&= \left(\frac{2+1}{2}\right)^2 \left(\frac{3+1}{3}\right)^2 \left(\frac{4+1}{4}\right)^2 \left(\frac{5+1}{5}\right)^2 \ldots \left(\frac{999+1}{999}\right)^2 \\
&= \left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^2 \left(\frac{5}{4}\right)^2 \left(\frac{6}{5}\right)^2 \ldots \left(\frac{1000}{999}\right)^2 \\
&= \frac{3^2}{2^2}\frac{4^2}{3^2}\frac{5^2}{4^2} \cdots \frac{1000^2}{999^2} \\
&= \frac{1000^2}{2^2} = 250000.
\end{aligned}$$

Translated by Andreas Munch Mikkelsen