The 25th Nordic Mathematical Contest 2011

Marking scheme - Guide lines

The notation from the solutions is used in the following.

Problem 1

Each of the following is worth 1 point:

- Prove that if all s_j are odd then for a given i a carry in column i will result in a carry in column $1000 - i$.
- Prove that if all s_j are odd then for a given i a carry from column i will result in a carry from column $1000 - i$.

The last 3 points are given for combining these observations to prove that it is not possible that all s_j are odd. One of these points can be given for a proof of a connection between the existence of carries in more than two columns. If there are small lacks in the argumentation, it will cost 1 point.

If no other points are given, the observation that there is no carry in column 0 and the observation with proof that there is a carry in column 500 if s_{500} is odd are worth totally 1 point.

Notice: If all s_i are odd the presence of a carry in column i is equivalent to $a_i + a_{1000-i}$ being even. While properties of columns are expressed above in terms of carries, equivalent expressions in terms of the parity of $a_i + a_{1000-i}$ are of course equally valid.

Problem 2

It is worth 4 point to prove the inequality $CD \geq \dfrac{4h}{BC} CE$ and worth 1 point to prove that equality happens exactly when E is the midpoint of BC or anything equivalent.

Of the 4 points each of the following parts is worth 1 point

- Reducing the problem by using a similarity, for example $\triangle BCD \sim \triangle BEA$.
- Reducing the problem by using a partial estimate, for example $AE \geq h$.

If there are lacks in the argumentation that can be repaired, 1 or 2 points are subtracted.

Problem 3

- Proving that $f(x) = 0$ or $f(x) = x^2$ for all $x \in \mathbb{R}$: 1 points.
- Proving that either $f(x) = 0$ for all $x \in \mathbb{R}$ or $f(x) = x^2$ for all $x \in \mathbb{R}$: 3 additional points.
  - Proving that if there exists an a such that $f(a) \neq 0$ then $y \neq 0$ implies that $f(a^2 \pm y)$ are not both 0 or something equivalent is worth 1 of these point.
  - Further deduce that $f(x) = x^2$ for $x \neq a^2$ or that $f(x) = 0$ implies $x = 0$ or $x = a^2$ is worth 1 more of these points.
- Proving that $f(x) = 0$ for all $x \in \mathbb{R}$ and $f(x) = x^2$ for all $x \in \mathbb{R}$ are both solutions to the equation: 1 point. (This point is given no matter what else has been done.)

Problem 4

Let S_n be the sum of the fractions.

- Proving that $S_2 = \frac{1}{2}$ and showing the intention of proving that $S_n - S_{n-1} = 0$ for all $n \geq 3$: 1 point.
- Precise description of the set A of terms in S_{n-1} that are not a part of S_n and the set B of terms in S_n that are not a part of S_{n-1}: 1 additional point.
- Proving that the relation $(a, b) \leftrightarrow \{(a, a+b), (b, a+b)\}$ is a bijection between terms in A and disjoint pairs of terms in B: 2 additional points. (1 point is subtracted if it is not discussed that terms corresponding to $a = b$ are absent from B.)
- Proving that the sum of a pair of terms in B that is assigned to a term in A is equal to this term: 1 additional point.