The 17$^{\text{th}}$ Nordic Mathematical Contest

Thursday April 3$^{\text{rd}}$ 2003

Problem 1

We place some stones on a rectangular chessboard with 10 rows and 14 columns. Afterwards we realize that there are an odd number of stones in each row and each column. If the squares of the chessboard are coloured alternatingly black and white (as is usual), show that the total number of stones on black squares is even. Note that a square may contain more than one stone.

Problem 2

Find all triplets (x, y, z) of integers such that

$$x^3 + y^3 + z^3 - 3xyz = 2003.$$

Problem 3

The equilateral triangle $\triangle ABC$ contains a point D such that $\angle ADC = 150°$. Prove that a triangle whose sides are $|AD|$, $|BD|$ and $|CD|$ is necessarily a right triangle.

Problem 4

Let $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ be the set of all real numbers except zero. Find all functions $f : \mathbb{R}^* \to \mathbb{R}^*$ satisfying

$$f(x) + f(y) = f(xyf(x + y))$$

for all $x, y \in \mathbb{R}^*$ such that $x + y \neq 0$.

Time permitted: 4 hours.
Each problem is worth 5 points.
Only writing and drawing utensils are allowed.