14th Nordic Mathematical Contest

March 30, 2000

Problem 1. In how many ways can the number 2000 be written as a sum of three positive, not necessarily different integers? (Sums like $1+2+3$ and $3+1+2$ etc. are the same.)

Problem 2. The persons P_1, P_1, $\ldots$, P_{n-1}, P_n sit around a table, in this order, and each one has a number of coins. In the start, P_1 has one coin more than P_2, P_2 has one coin more than P_3, etc., up to P_{n-1} who has one coin more than P_n. Now P_1 gives one coin to P_2, who in turn gives two coins to P_3 etc., up to P_n who gives P_1 n coins. Now the process continues in the same way: P_1 gives $n+1$ coins to P_2, P_2 gives $n+2$ coins to P_3; in this way the transactions go on until someone has not enough coins, i.e. a person no more can give away one coin more than he just received. At the moment when the process comes to an end in this manner, it turns out that there are to neighbors at the table such that one of them has exactly five times as many coins as the other. Determine the number of persons and the number of coins circulating around the table.

Problem 3. In the triangle ABC, the bisector of angle B meets AC at D and the bisector of angle C meets AB at E. The bisectors meet each other at O. Furthermore, $OD = OE$. Prove that either ABC is isosceles or $\angle BAC = 60°$.

Problem 4. The real-valued function f is defined for $0 \le x \le 1$, $f(0) = 0$, $f(1) = 1$, and

$$\frac{1}{2} \le \frac{f(z) - f(y)}{f(y) - f(x)} \le 2$$

for all $0 \le x < y < z \le 1$ with $z - y = y - x$. Prove that

$$\frac{1}{7} \le f\left(\frac{1}{3}\right) \le \frac{4}{7}.$$

Each problem is worth 5 points.

Time allowed: 4 hours.