

# THE GEORG MOHR CONTEST 2025

## First round

Tuesday, 26 November 2024

*Duration: 90 minutes*

*Aids allowed: none*

*Tick the answers on the included answer sheet*

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

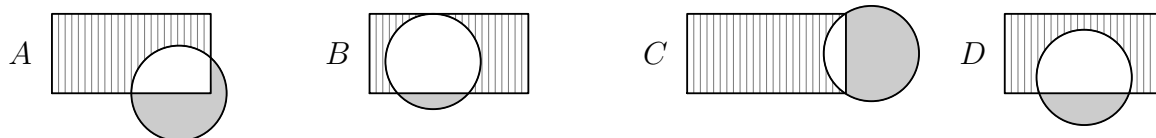
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### MULTIPLE CHOICE PROBLEMS

To each of the problems 1 – 10 there are five options, A, B, C, D and E.  
One of these options is the correct answer.

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1. The four figures show the same rectangle and the same circle, but placed differently with respect to each other.

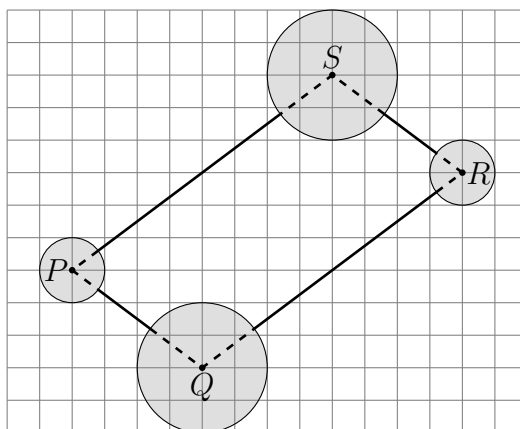


In which figure is the area of the hatched area minus the area of the grey area the largest?

- A) A    B) B    C) C    D) D    E) it is equally big on all figures
2. Gerth wants to build a model railway. He has lots of track pieces of lengths 7 cm and 11 cm available. What is the least number of pieces he needs to build a railway which is exactly 200 cm long?
- A) 18    B) 20    C) 22    D) 24    E) 26
3. The numbers 1, 2, ..., 9 are written on a blackboard. Nikolaj changes one of the numbers by appending a 0 and two other numbers by prepending a 1. The sum of the numbers is now 119. Which of the original numbers had a 0 appended?

A) 3    B) 6    C) 7    D) 8    E) it cannot be determined

4. The figure shows a map of a playground where narrow beams have been installed so one can practice keeping one's balance by walking the path  $PQRS$ . The squares on the map have sidelengths of 1 meter. The circles are platforms, two of radius 1 meter and two of radius 2 meters. What is the total length of the part of the path which runs outside the platforms?



- A) 16 m    B) 18 m    C) 20 m    D) 24 m    E) 26 m
5. Alma rolls an 8-sided die with the numbers  $1, 2, \dots, 8$  on the sides, and Bertha rolls a 20-sided die with the numbers  $1, 2, \dots, 20$  on the sides. What is the probability that they roll the same number?

- A)  $\frac{1}{8}$     B)  $\frac{8}{20}$     C)  $\frac{1}{12}$     D)  $\frac{1}{20}$     E)  $\frac{1}{28}$

6. A  $2 \times 2025$  board must be covered by pieces of this shape:



In how many ways can this be done so that the pieces do not overlap?

- A) 1    B) 2025    C)  $2 \cdot 3 \cdot 2025$     D)  $2025^2$     E)  $2^{2025/3}$
7. It is known that the four numbers  $a, b, c$  and  $d$  satisfy  $a < b < c < d$  and  $\frac{1}{c} < \frac{1}{b} < \frac{1}{a} < \frac{1}{d}$ . How many of the four numbers are negative?

- A) 0    B) 1    C) 2    D) 3    E) it cannot be determined
8. On one side of an infinitely long road there are houses numbered

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

All the houses are either red or blue. A house is blue if its two neighbouring houses have different colours, otherwise it is red. House number 492 is blue. What can be said with certainty?

- A) house number 2024 is red    B) house number 2024 is blue  
 C) house number 2025 is red    D) house number 2025 is blue  
 E) one cannot be certain of any of the above

9. The numbers 1, 2, ..., 9 must be placed in the nine squares of the large square. The numbers in the top row have the sum 19, and the numbers in the bottom row have the sum 15. The numbers in the left column have the sum 7, and the numbers in the right column have the sum 14. Which number must be put in the square with the question mark?

|    |   |   |    |
|----|---|---|----|
|    | 7 |   | 14 |
| 19 |   |   |    |
|    |   |   |    |
| 15 |   | ? |    |

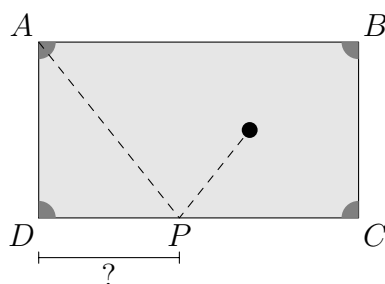
- A) an odd number    B) 4    C) 6    D) 8    E) it cannot be determined
10. An urn contains two black candies and one white candy. Now Georg puts candies in the urn using the following principle: He picks a random candy from the urn, then puts this candy and one more of the same colour back again. He does that three times so that in the end there are six candies in the urn. What is the probability that there ends up being three candies of each colour?

- A)  $\frac{1}{2}$     B)  $\frac{1}{3}$     C)  $\frac{1}{4}$     D)  $\frac{1}{5}$     E)  $\frac{1}{6}$

### ANSWER PROBLEMS

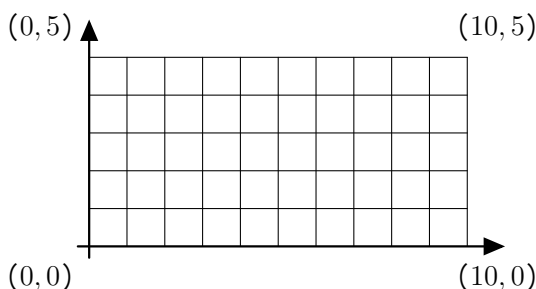
The answer to each of the problems 11 – 20 is a positive integer

11. On a list of the 100 most frequently used words in a certain language there are 70 words that contain the letter A and 60 words that contain the letter U, while 20 of the words contain neither A or U. How many of the words contain both A and U?
12. Georg chooses a two-digit positive integer  $n$ . When he divides 1010 by  $n$ , he gets a remainder of 2. Which remainder will he get when he divides 2025 by  $n$ ?
13. Paula has a bag containing three E's, three G's, three H's, three M's, three O's and three R's. She draws letters from the bag one at a time without looking. How many letters must she at least draw to be certain that she can write GEORG MOHR using the drawn letters?
14. Aisha is playing pool on a pool table of length 282 cm and width 155 cm. The ball lies equally far from the two long sides and 96 cm from the side  $BC$ . She wants to pot the ball in the pocket at  $A$  by shooting it into the long side  $CD$  at the point  $P$  as shown.



How many cm must there be between  $D$  and  $P$  if Aisha is to hit the pocket at  $A$ ?

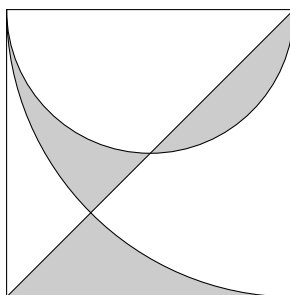
15. In a coordinate system a rectangle with the corners  $(0,0)$ ,  $(10,0)$ ,  $(10,5)$  and  $(0,5)$  is divided into 50 small unit squares. For each of the small squares the *value* is defined as the sum of the four corners' first and second coordinates. Thus e.g. the unit square with the corners  $(3,2)$ ,  $(4,2)$ ,  $(4,3)$  and  $(3,3)$  has the value  $3+2+4+2+4+3+3+3 = 24$ . How many distinct values appear in total?



16. Determine the value of

$$\frac{20202^2 \cdot 80808^2}{40404^4}.$$

17. The numbers from 1 to 2025 are written in a row in random order. Randi appends 1000 numbers, one at a time, to the end of the row in this way: She looks at the last 2025 numbers in the row and then adds the median of those 2025 numbers. She does that 1000 times. How many distinct numbers can there maximally be among the 1000 she appends? (The median of 2025 numbers is the number in the middle when the numbers are ordered by size.)
18. The three ducklings Zip, Zap and Zup each have a favorite number, which is an integer greater than 1. The three favorite numbers are distinct. The ducklings now take turns saying some number, and if their favorite number divides the mentioned number, they must raise a wing. Zip says 20, and two wings are raised. Zap says 21, and one wing is raised. Zup says 70, and all three of them raise a wing. What is the largest possible sum of the three favorite numbers?
19. Inside a square with side length 10 a half circle, a quarter circle and a diagonal have been drawn.



What is the area of the grey region?

20. About the positive integers  $A$  and  $B$  one knows that the sum of the digits in  $A$  is 2025, and the sum of the digits in  $B$  is 60. What is the least possible sum of the digits in  $A + B$ ?

*Sponsors: Undervisningsministeriet, Novo Nordisk Fonden, LEO Fondet, Jobindex, Institut for Matematiske Fag KU, Institut for Matematik og Datalogi SDU, Institut for Matematik AU, Institut for Matematiske Fag AAU, Georg Mohr Fonden.*