The Georg Mohr Contest 2025
Second Round

Tuesday, 14 January 2025 at 9–13

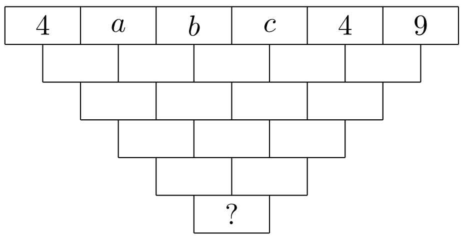
Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. Georg and his mother play a game with coins. Initially, Georg places some number of coins on the table. Then his mother places twice as many coins. Then Georg places so many coins that he has in total placed twice as many coins as his mother. Then his mother places so many coins that she has in total placed twice as many coins as Georg. Finally, Georg places 240 coins and has now in total placed twice as many as his mother.

How many coins did Georg place initially?

Problem 2. In the boxes in the top row of the figure, the numbers $4, a, b, c, 4, 9$ are written as shown in the figure, where a, b and c are among the numbers $0, 1, 2, \ldots, 9$. In each box lying below two other boxes, one writes the last digit of the sum of the two numbers in the boxes above.

Which numbers may be written in the bottommost box?

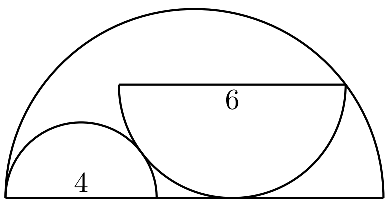


Problem 3. Alma and Bertha play the following game. Initially they are standing by a blank board. They take turns writing one of the numbers $1, 2, \ldots, 2025$ on the board. They may not write a number which is already written on the board. Alma begins. One loses the game by being the first to write a number so that 2025 divides the product of all the numbers on the board.

Which player can find a strategy ensuring she will win?

Problem 4. Two semicircles with diameters 4 and 6 lie inside a third semicircle as shown in the figure. The two small semicircles are tangent to each other, and their diameters are parallel.

What is the diameter of the large semicircle?



Problem 5. Georg plays the following game. He chooses a positive integer n, which he writes on a board. In each move Georg does the following: If the number m written on the board is even, he replaces m by $\frac{m}{2}$. If the number m written on the board is odd, he replaces m by $m^2 + 3$. Georg wins if he after a finite number of moves writes the number n on the board again.

For which positive integers n does Georg win?

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