# The Viking Battle - Part 1 2015
## Version: English

**Problem 1** Let $n \geq 2$ be an integer, and let $A_n$ be the set

$$A_n = \{2^n - 2^k | k \in \mathbb{Z}, 0 \leq k < n\}.$$

Determine the largest integer $M_n$ that cannot be written as the sum of one or more not necessarily distinct elements of $A_n$.

**Problem 2** Define the function $f : (0, 1) \to (0, 1)$ by

$$f(x) = \begin{cases} x + \frac{1}{2} & , \ x < \frac{1}{2} \\ x^2 & , \ x \geq \frac{1}{2} \end{cases}$$

Let $a_0$ and $b_0$ be two reel numbers such that $0 < a_0 < b_0 < 1$. We define the sequences $a_n$ and $b_n$ by $a_n = f(a_{n-1})$ and $b_n = f(b_{n-1})$ for all $n = 1, 2, 3, \ldots$.

Show that there exists a positive integer $n$ such that

$$(a_n - a_{n-1}) \cdot (b_n - b_{n-1}) < 0.$$

**Problem 3** Let $\Omega$ and $O$ be the circumcircle and the circumcentre of an acute triangle $ABC$ with $AB > BC$. The angle bisector of $\angle ABC$ intersects $\Omega$ at $M \neq B$. Let $\Gamma$ be the circle with diameter $BM$. The angle bisectors of $\angle AOB$ and $\angle BOC$ intersect $\Gamma$ at points $P$ and $Q$ respectively. The point $R$ is chosen on the line $PQ$ such that $BR = MR$. Prove that $BR \parallel AC$. (Here we always assume that an angle bisector is a ray.)