The Viking Battle - Part 1 2014
Version: English

Problem 1 Let $\mathbb{N}$ be the set of positive integers. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$m^2 + f(n) \mid mf(m) + n$$

for all positive integers m and n.

Problem 2 Let ω be the circumcircle of triangle ABC. Denote by M and N the midpoints of the sides AB and AC, respectively, and denote by T the midpoint of the arc BC of ω not containing A. The circumcircles of the triangles AMT and ANT intersect the perpendicular bisectors of AC and AB at points X and Y, respectively. Assume that X and Y lie inside the triangle ABC. The lines MN and XY intersect at K. Prove that $KA = KT$.

Problem 3 A crazy physicist discovered a new kind of particle which he called an *imon*, after some of them mysteriously appeared in his lab. Some pairs of imons are *entangled*, and each imon can participate in many entanglement relations. The physicist has found a way to perform the following two kinds of operations with these particles, one operation at a time.

(i) If some imon is entangled with an odd number of other imons in his lab, then the physicist can destroy it.

(ii) At any moment, he may double the whole family of imons in his lab by creating a copy I' of each imon I. During this procedure, the two copies I' and J' become entangled if and only if I and J are entangled, and each copy I' becomes entangled with its original imon I; no other entanglements occur or disappear at this moment.

Prove that the physicist may apply a sequence of such operations resulting in a family of imons, no two of which are entangled.