The 40th Nordic Mathematical Contest

27 March 2026

Time allowed: 4 hours. Each problem is worth 7 points.
Only writing and drawing tools are allowed.

Problem 1 Let $n \geq 3$ be an integer. There are n knights sitting around a round table. On the table there are n candles, such that between each pair of adjacent knights there is a single candle. Some candles (possibly all, but possibly also none) are lit. For $i \in \{0,1,2\}$, let m_i be the number of knights sitting next to exactly i lit candles.

Find the smallest possible value of $m = \max\{m_0, m_1, m_2\}$ in terms of n.

Problem 2 Consider the system of equations:

$$\begin{cases} x^2 = y + 1, \\ xy = x + y. \end{cases}$$

Show that if $(x, y) = (x_0, y_0)$, where x_0 and y_0 are real numbers, is a solution to the above system of equations, then there is also a solution $(x, y) = (x_1, y_1)$, where x_1 and y_1 are real numbers and $y_1 x_0 = 1$.

Problem 3 Let $ABCD$ be a convex quadrilateral such that $BA = BC$. The internal angle bisectors of $\angle DBA$ and $\angle CBD$ intersect the perpendicular bisectors of AD and CD in E and F, respectively.

Prove that the circumcircles of $\triangle DAC$ and $\triangle DEF$ are tangent.

Note: A convex quadrilatral is a quadrilateral where all angles are smaller than $180°$.

Note: The internal angle bisector of an angle is the line segment that divides the angle into two equal parts.

Problem 4 A pair of positive integers (a, b) is *good* if all the fractions

$$\frac{a}{b}, \frac{a+1}{b+1}, \dots, \frac{a+9}{b+9}$$

are integers.

a) Prove that there are only finitely many good pairs (a, b) with $b < a < b^9$.

b) Prove that there are infinitely many good pairs (a, b) with $b < a < b^{10}$.