# The 30th Nordic Mathematical Contest
# Marking scheme

**Problem 1**

1. Proving $a_{2016} = \ldots = a_{1009}$: 3 points (alternatively showing that at least half of the sequence is constant).

2. Proving that for all $i < 1009$ there is some $j > 1008$ such that $a_i = a_j$: 3 points (These point may be awarded even if the first 3 points are not).

   Additive partial points of 1 and 2 are awarded as follows

   - Proving that $a_i = a_j$ for some pair $i \neq j$: 1 point.
   - Rewriting and concluding $i + j \mid i(a_i - a_j)$ for all $i, j$ or something similar: 1 point. Proving that $i + j \mid a_i - a_j$ when $\gcd(i, j) = 1$: additionally 1 point.

3. Concluding that the solutions are exactly the constant sequences: 1 point.

*Remark*: Stating that the solutions are exactly the constant sequences without proof: 0 points.

**Problem 2** Partial points are awarded as follows and only 1 and 2 are additive.

1. Proving $\angle BCA = \angle ACD$: 1 point.

2. Constructing a point $E$ on the segment $CD$ such that $DE = AD$ and stating that $\triangle BCE$ and $\triangle ADE$ are isosceles: 1 point.

3. Proving that $BE$ and $AC$ are perpendicular: 4 points.

4. Proving that $AC$ is the perpendicular bisector of $BE$: 5 points.

5. Proving that $AE = AB$: 6 points.

*Remark*: Stating that $\angle CDA = 60°$ without proof: 0 points.

**Problem 3** The points from 1, 2 and 3 are additive.

1. Proving that when $a \neq \frac{1 \pm \sqrt{5}}{2}$ there is no such function: 5 points.
   Partial points are awarded as follows and are NON-additive:

   - Proving that $f(x) = -\frac{1}{a}x$: 4 points
     Partial points are awarded as follows and are NON-additive:
     - Proving that $f(f(f(x)) - f(x)) = f(x)$ or $f(f(f(x)) - f(x)) = f(f(x)) + af(x)$: 1 point.

    - Proving that $f(x) = f(f(x)) + af(x)$: 2 points.

- Proving that $f(x) = -\frac{1}{a}x$ is not a solution when $a \neq \frac{1\pm\sqrt{5}}{2}$: 1 point.
- Proving that $f(x_0) = -\frac{1}{a}x_0$ for some nonzero $x_0 \in \mathbb{R}$: 1 point. Proving $f(f(x_0)) = \frac{1}{a^2}x_0$ for some nonzero $x_0 \in \mathbb{R}$: 2 points. Proving both for the same $x_0$: 3 points.

2. Proving that $f(x) = -\frac{1}{a}x$ where $a = \frac{1\pm\sqrt{5}}{2}$ satisfy *(i)*: 1 point.

3. Proving that $f(x) = -\frac{1}{a}x$ where $a = \frac{1\pm\sqrt{5}}{2}$ satisfy *(ii)*: 1 point.

*Remark*: Stating that $a = \frac{1\pm\sqrt{5}}{2}$ are the only solutions without proof: 1 point if no other points are awarded.

**Problem 4** The points from 1 and 2 are additive.

1. Establishing the lower bound of 2016 bridges: 4 points.
   Partial points are awarded as follows and are NON-additive:

   - Proving that if two islands with exactly two bridges are connected by a bridge, then the two bridges not connecting them must go to the same island: 1 point.
   - Establishing inequalities and equations capable of yielding the lower bound of 2016 bridges only by use of the inequalities and equations: 3 point.

   If the lower bound is established under the assumption that no two islands with exactly two bridges are connected by a bridge, 1 point is subtracted.

2. Proving the existence of a solution using 2016 bridges: 3 points.
   Partial points are awarded as follows:

   - Constructing a solution using 2016 bridges that satisfies the conditions: 2 points. (A construction with at most 2100 bridges: 1 point).
   - Proving that the given construction with 2016 bridges is correct: additionally 1 point.

*Remark*: Stating that the minimal number of bridges is 2016 without proof: 1 point if no other points are awarded.