# The 26th Nordic Mathematical Contest

## Tuesday, 27 March 2012

## English Version

*The time allowed is 4 hours. Each problem is worth 5 points. The only permitted aids are writing and drawing tools.*

Problem 1. The real numbers $a, b, c$ are such that $a^2 + b^2 = 2c^2$, and also such that $a \neq b, c \neq -a, c \neq -b$. Show that

$$\frac{(a+b+2c)(2a^2-b^2-c^2)}{(a-b)(a+c)(b+c)}$$

is an integer.

Problem 2. Given a triangle $ABC$, let $P$ lie on the circumcircle of the triangle and be the midpoint of the arc $BC$ which does not contain $A$. Draw a straight line $l$ through $P$ so that $l$ is parallel to $AB$. Denote by $k$ the circle which passes through $B$, and is tangent to $l$ at the point $P$. Let $Q$ be the second point of intersection of $k$ and the line $AB$ (if there is no second point of intersection, choose $Q = B$). Prove that $AQ = AC$.

Problem 3. Find the smallest positive integer $n$, such that there exist $n$ integers $x_1, x_2, \ldots, x_n$ (not necessarily different), with $1 \leq x_k \leq n,\ 1 \leq k \leq n$, and such that

$$x_1 + x_2 + \cdots + x_n = \frac{n(n+1)}{2}, \quad \text{and} \quad x_1 x_2 \cdots x_n = n!,$$

but $\{x_1, x_2, \ldots, x_n\} \neq \{1, 2, \ldots, n\}$.

Problem 4. The number 1 is written on the blackboard. After that a sequence of numbers is created as follows: at each step each number $a$ on the blackboard is replaced by the numbers $a-1$ and $a+1$; if the number 0 occurs, it is erased immediately; if a number occurs more than once, all its occurrences are left on the blackboard. Thus the blackboard will show 1 after 0 steps; 2 after 1 step; $1, 3$ after 2 steps; $2, 2, 4$ after 3 steps, and so on. How many numbers will there be on the blackboard after $n$ steps?