

# 20<sup>th</sup> Nordic Mathematical Contest

Thursday March 30, 2006

English version

Time allowed: 4 hours. Each problem is worth 5 points.

**Problem 1.** Let  $B$  and  $C$  be points on two fixed rays emanating from a point  $A$  such that  $AB + AC$  is constant.

Prove that there exists a point  $D \neq A$  such that the circumcircles of the triangles  $ABC$  pass through  $D$  for every choice of  $B$  and  $C$ .

**Problem 2.** The real numbers  $x$ ,  $y$  and  $z$  are not all equal and fulfill

$$x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x} = k$$

Determine all possible values of  $k$ .

**Problem 3.** A sequence of positive integers  $\{a_n\}$  is given by

$$a_0 = m \quad \text{and} \quad a_{n+1} = a_n^5 + 487 \quad \text{for all } n \geq 0$$

Determine all values of  $m$  for which the sequence contains as many square numbers as possible.

**Problem 4.** The squares of a  $100 \times 100$  chessboard are painted with 100 different colours. Each square has only one colour and every colour is used exactly 100 times.

Show that there exists a row or a column on the chessboard in which at least 10 colours are used.

*Only writing and drawing sets are allowed*