Nordic Mathematical Contest 1997

1. For any set A of positive integers, let n_A denote the number of triples (x, y, z) of elements of A such that $x < y$ and $x + y = z$. Find the maximum value of n_A given that A contains seven distinct elements.

2. Let $ABCD$ be a convex quadrilateral. Assume that there exists an internal point P of $ABCD$ such that the areas of the triangles ABP, BCP, CDP and DAP are all equal. Prove that at least one of the diagonals of the quadrilateral bisects the other.

3. Assume A, B, C, D are four distinct points in the plain. Three of the segments AB, AC, AD, BC, BD, CD have length a. The other three have length $b > a$. Find all possible values of the ration b/a.

4. Let f be a function defined on $\{ 0, 1, 2, ... \}$ such that

   $f(2x) = 2f(x)$

   $f(4x + 1) = 4f(x) + 3$

   $f(4x - 1) = 2f(2x - 1) - 1$

   Prove that f is injective (if $f(x) = f(y)$, then $x = y$).