# VIII Nordic Mathematical Contest

## Thursday 17th March 1994

Time permitted: 4 hours

Each problem is worth 5 points.

**Problem 1**

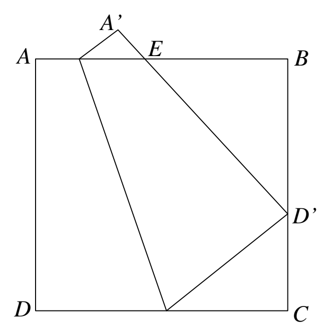
Let $O$ be a point in the interios of an equilateral triangle $ABC$ with sides of length $a$. The lines $AO$, $BO$, and $CO$ intersect the edges in the points $A_1$, $B_1$ og $C_1$. Prove that

$$|OA_1| + |OB_1| + |OB_2| < a.$$

**Problem 2**

A finite set $S$ of points in the plane with integer coordinates is a *two-neighbor-set* if for each point $(p, q)$ in $S$, exactly two of the points $(p+1, q)$, $(p-1, q)$, $(p, q+1)$ and $(p, q-1)$ are in $S$. For which $n$ does there exist a two-neighbor-set containing exactly $n$ points?

**Problem 3**




A square sheet $ABCD$ is folded by placing the corner $D$ at a point $D'$ on $BC$ (see figure). Then, $AD$ is moved to $A'D'$ which intersects $AB$ in $E$. Prove that the circumference of $EBD'$ is half as long as the circumference of the square.

**Problem 4**

Find all positive integers $n < 200$ such that $n^2 + (n+1)^2$ is a perfect square.