

# The 21st Nordic Mathematical Contest

March 29, 2007

English version

*Time allowed: 4 hours. Each problem is worth 5 points. Only writing and drawing sets are allowed.*

## Problem 1

Find *one* solution in positive integers to the equation

$$x^2 - 2x - 2007y^2 = 0.$$

## Problem 2

A triangle, a line and three rectangles, with one side parallel to the given line, are given in such a way that the rectangles completely cover the sides of the triangle. Prove that the rectangles must completely cover the interior of the triangle.

## Problem 3

The number  $10^{2007}$  is written on a blackboard. Anne and Berit play a game where the player in turn makes one of two operations:

(i) replace a number  $x$  on the blackboard by two integer numbers  $a$  and  $b$  greater than 1 such that  $x = ab$ ;

(ii) erase one or both of two equal numbers on the blackboard.

The player who is not able to make her turn loses the game. Who will win the game if Anne begins and both players act in an optimal way?

## Problem 4

A line through a point  $A$  intersects a circle in two points,  $B$  and  $C$ , in such a way that  $B$  lies between  $A$  and  $C$ . From the point  $A$  draw the two tangents to the circle, meeting the circle at points  $S$  and  $T$ . Let  $P$  be the intersection of the lines  $ST$  and  $AC$ . Show that  $AP/PC = 2 \cdot AB/BC$ .