

THE GEORG MOHR CONTEST 2017

First round

Tuesday, November 15 2016

Duration: 90 minutes

Aids allowed: none

Tick the answers on the included answer sheet

REMEMBER that there are 20 questions to be answered in a total of 90 minutes. If you cannot solve a problem, it is a good idea to skip it and go on to the next problem.

MULTIPLE CHOICE PROBLEMS

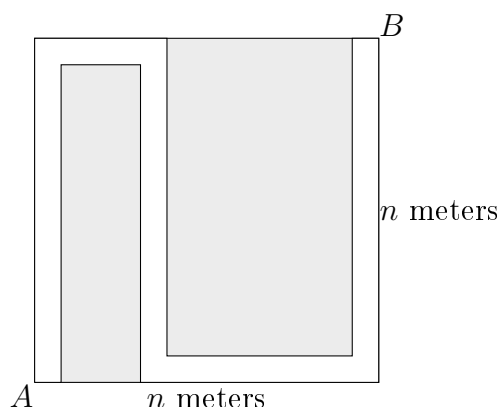
To each of the problems 1 – 10 there are five options A, B, C, D, and E.

One of these options is the correct answer.

1. A fly says to itself: Tomorrow is Wednesday, and then I will be twice as old as I was last Friday. How many days old is the fly today?

A) 5    B) 7    C) 8    D) 9    E) 11

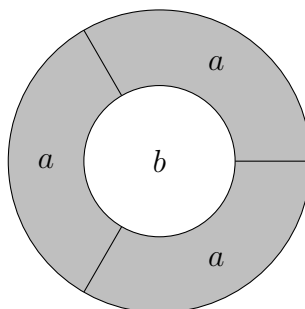
2. In an exhibition area measuring  $n$  meters by  $n$  meters, the audience is led through the exhibition along a 1 meter wide path from the corner  $A$  to the corner  $B$  as shown in the figure.



How many  $m^2$  is left for the exhibition when the area of the path is deducted?

A)  $n^2 - 3n + 1$     B)  $n^2 - 4n + 3$     C)  $n^2 - n - 5$     D)  $n^2 - 4n$     E)  $n^2 - 3n - 10$

3. The figure shows two circles with radii 1 and 2, respectively. The area of each of the gray areas is  $a$ . The area of the white center circle is  $b$ . What is  $\frac{a}{b}$ ?



A)  $\frac{3}{4}$     B)  $\frac{3}{2}$     C)  $\frac{8}{3}$     D)  $\frac{4}{3}$     E) 1

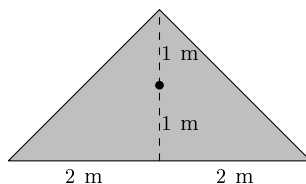
4. Three positive numbers  $x$ ,  $y$  and  $z$  are known to satisfy  $2x > 3y > 4z$ . Which of these can be deduced?

A)  $x < y < z$    B)  $x > 2y > 3z$    C)  $3x > 4y > 5z$    D)  $3x > 5y > 7z$    E)  $x > 3y > 5z$

5. Georg flips a coin repeatedly. After each flip he writes an A if he got heads, and a B if he got tails, each time writing the letter after the previous one. After four flips he has a four letter word. Which of the following words is most likely?

A) ABBA   B) BBAB   C) ABAB   D) AAAA   E) these are equally probable

6. A crane has a flat, triangular foot. The figure shows the foot as seen from above. The foot can rotate freely around a vertical axis mounted in the point indicated. The region of the floor which the foot may cover is painted yellow. What is the area of the yellow region?



A)  $6 \text{ m}^2$    B)  $15 \text{ m}^2$    C)  $3\pi \text{ m}^2$    D)  $16 \text{ m}^2$    E)  $5\pi \text{ m}^2$

7. Anders, Benjamin, Carla and Dagmar each either always tell the truth or always lie. They say:

Anders: *Dagmar is lying.*

Benjamin: *Carla is lying.*

Dagmar: *One of Benjamin and Carla tells the truth, the other is lying.*

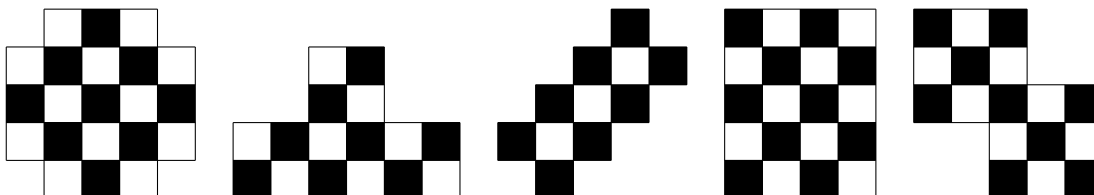
Who do we know with certainty always lies?

A) Anders   B) Benjamin   C) Carla   D) Dagmar   E) everyone

8. A charming rectangular plaza, measuring 44 meters on one side and 88 meters on the other side, is paved with beautiful square tiles all of the same size. Georg reveals that the number of tiles is one of the following five numbers. Which is it?

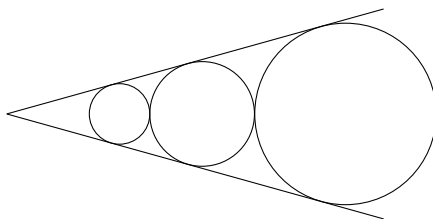
A) 2000   B) 8000   C) 10000   D) 11000   E) 20000

9. A one-man game is played on a board with black and white squares. In each move one must pick either a row or a column and then change all black squares to white and all white squares to black in the chosen row/column. One wins by making the board entirely black or entirely white in at most four moves. With how many of the following boards is it possible to win?



A) 1   B) 2   C) 3   D) 4   E) 5

10. The figure shows three circles which are all tangent to the two lines. The middle circle is furthermore tangent to the other two circles.



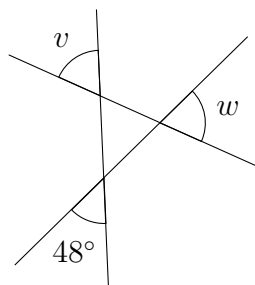
The small circle has radius 2 and the large circle has radius 6. What is the radius of the middle circle?

- A) 3    B)  $2\sqrt{3}$     C)  $\frac{10}{3}$     D)  $3\sqrt{2}$     E) 4

### ANSWER PROBLEMS

The answer to each of the problems 11 – 20 is a positive integer

11. What is the sum of angles  $v$  and  $w$  in degrees?



12. A treasure is locked behind seven gates. The keys to the first and last gate weigh the same. The keys to the second and sixth gate each weigh twice as much as the key to the first gate. The keys to the third and fifth gate each weigh twice as much as the key to the second gate. And the key to the middle gate weighs twice as much as the key to the third gate. Altogether, the seven keys weigh 880 grams. How many grams does the heaviest key weigh?

13. The number  $a$  is known to satisfy

$$2017a^{2017} + 100 = 110.$$

What is

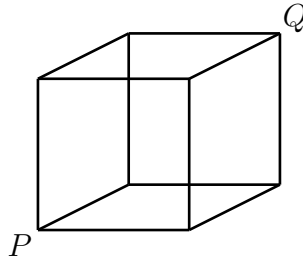
$$2017(-a)^{2017} + 100 ?$$

14. Ane, Benny and Charlotte each think of a positive number. If one removes the last digit of Ane's number one gets Benny's number. If one removes the last digit of Benny's number one gets Charlotte's number. The sum of the three numbers is 395. What number is Ane thinking of?

15. Which positive integer  $n$  satisfies

$$n(n+3)(n+6) \cdots (n+297)(n+300) = (200-n)(203-n)(206-n) \cdots (497-n)(500-n) ?$$

16. The figure shows a cube made of 12 pieces of wire. An ant crawls along the wire from the point  $P$  to the point  $Q$ . On the way, it visits each of the cube's other corners exactly once. How many routes are possible?



17. The sum of 81 consecutive integers is  $3^8$ . What is the middle of the 81 numbers?

18. Peter and his friends have 100 large chocolate frogs. They share them equally among themselves. Peter immediately eats one of his frogs and then one more, but at some point he feels sick and stops eating more. He computes that he has eaten exactly one quarter of his frogs. How many frogs does Peter have left?

19. Every morning Esben uses a red pen to write his weight in grams in his notebook. He also computes the average of today's weight and the two preceding days' weights and writes this number in blue. Every Sunday he computes the average of the entire week's red numbers and the average of the entire week's blue numbers.

If one assumes that Esben's weight never changes more than 700 grams from day to day, what is the largest possible difference in grams between the red and the blue week averages?

20. A circle is inscribed in the right triangle  $ABC$ , with the right angle at  $C$ . The circle touches the sides of the triangle at the points  $P$ ,  $Q$  and  $R$ . Angle  $Q$  in triangle  $PQR$  is  $74^\circ$ . How many degrees is angle  $B$ ?

