THE GEORG MOHR CONTEST 2012

First round

15 November 2011

Duration: 60 minutes
Aids permitted: none
Answer by ticking the enclosed answering sheet.

1. What is the value of the number

$$(77 + 7777 + 777777 + 77777777) - (7 + 777 + 77777 + 7777777) \ ?$$

A) 14071407 B) 70707070 C) 700000 D) 7 E) 0

2. The traditional alphabet in Mohristan consists of the six basic symbols shown below and all the symbols obtained from them by inverting them laterally and/or turning them upside down. In total, how many different symbols are there in the alphabet?

A) 18 B) 24 C) 12 D) 15 E) 20

3. Emma collects ice-cream sticks, beer-bottle caps and stickers. When calculating how much her collection is worth, she usually assumes that four ice-cream sticks equal seven beer-bottle caps and that ten stickers equal three beer-bottle caps. How many stickers equal six ice-cream sticks?

A) 35 B) 42 C) 14 D) 32 E) 18

4. The nine balls shown are to be arranged in numerical order (with number 1 leftmost) by means of as few operations as possible. Each operation consists in taking a ball and moving it to the right of all the others. What is the least sufficient number of operations?

A) 4 B) 5 C) 6 D) 8 E) 9

5. Ole chooses five of the six numbers 2, 3, 5, 7, 11 and 13 and calculates their product. What is the next largest result he can get?

A) 10010 B) 13013 C) 10110 D) 2310 E) 2735

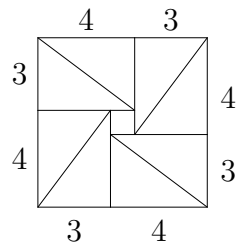
6. Happiness researchers investigate how happy people become by owning different things. They measure happiness on a scale from 0.00 to 1.00. They have found that if one thing gives a happiness x and another thing gives the happiness y, then one achieves a total happiness $x + y - xy$ if one owns both things. How happy will one become by owning both a thing with happiness value 0.30 and a thing with happiness value 0.50?

A) 0.08 B) 0.80 C) 0.65 D) 0.70 E) 1.50

7. A square is intersected by a straight line and thus divided into two parts. Which of the following possibilities *cannot* occur?

A) a triangle and a pentagon B) two quadrilaterals
C) two triangles D) a triangle and a quadrilateral
E) a pentagon and a quadrilateral

8. How long is the longest walk one can take along the paths in the figure when one must not walk the same stretch more than once? The walk must begin and end at the same place.



A) 41 B) 44 C) 50 D) 52 E) 58

9. The numbers a, b, c and d are positive integers with $0 < a < b < c < d$ which satisfy $a^2 + b^2 + c^2 + d^2 = 50$. What is the value of the number c?

A) 3 B) 4 C) 5 D) 6 E) it cannot be decided

10. Marie and Louise have made a secret code. Each letter in the alphabet corresponds to a one-digit or two-digit number. The message *skatten er skjult i haven* is coded to

213427777176 1726 21338768977 96 824237176

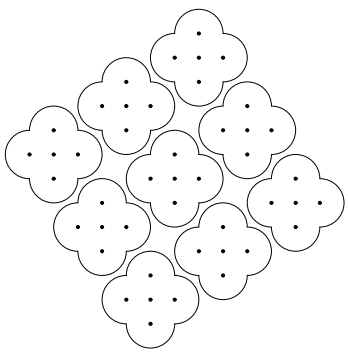
What is the code for the letter u?

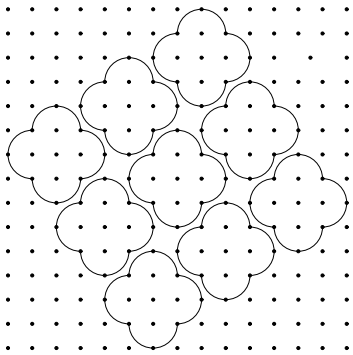
A) 6 B) 68 C) 76 D) 87 E) 7

11. Snif always loses to Snaf and Snuf in long jumping. Snyf always wins over Snuf. Snøf loses to Snaf but wins over Snuf. Who of the five jumps the longest?

A) Snuf B) Snaf C) Snyf D) Snøf E) it cannot be decided

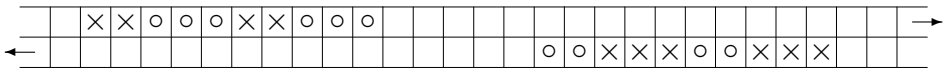
12. The famous "Great-Grandmother's Biscuits" are sold in packs of nine biscuits. The characteristic "flower shape" outline of the biscuits is formed by semicircles with radius 1. The figure on the right shows how the biscuits lie in the pack. What is the closest two biscuits get to each other?





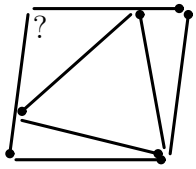
A) $\sqrt{2}-1$ B) $2\sqrt{3}-2$ C) $\sqrt{5}-2$ D) $3-\sqrt{3}$ E) $2\pi-6$

13. The figure shows a starting set-up of game pieces of two types $\times$ and $\circ$. Each time exactly one minute has passed, all the pieces in the upper row move one square to the right and all those in the lower row one square to the left. We say that two pieces that are situated vertically above one another make a "lucky pair" if they are of the same type. Of course the number of lucky pairs depends on where the pieces stand; in the beginning this number is 0. What is the largest number of lucky pairs that will occur?



A) 2 B) 6 C) 7 D) 4 E) 8

14. Seven equally long matches are to be assembled into a figure in the style of the one shown. The matches must just touch one another. How large does the angle indicated with the question mark become?

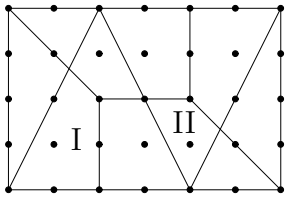


A) 100° B) 96° C) 108° D) 80° E) 120°

15. Which of the following numbers is the largest?

A) $\sqrt{408}$ B) 20 C) $\sqrt{100}+\sqrt{308}$ D) $\frac{\sqrt{816}}{2}$ E) $2\sqrt{204}$

16. A rectangle with sidelengths 4 and 6 is cut into ten parts as shown. What is the total area of the domains I and II?



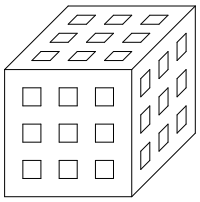
A) 4 B) 5 C) $4\frac{2}{3}$ D) 10 E) $4\frac{1}{3}$

17. In how many ways can the number 55 be written as a sum of at least two consecutive integers? (Negative numbers and 0 may enter the sum.)

A) 1 B) 2 C) 3 D) 6 E) 7

18. In a cube with sidelength $2n+1$ channels are drilled in all three directions all the way through the cube parallel to its sides. Every face of the cube has n^2 channels opening into it. The channels have a square cross section with sidelength 1, and the separation of the channels is 1. What is the volume of what is left?

The figure shows the case $n = 3$.

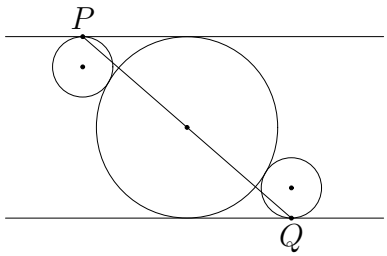


A) $4n^3 + 9n^2 + 6n + 1$ B) $4n^3 + 7n^2 + 4n + 1$
C) $6n^3 + 7n^2 + 4n + 3$ D) $13n^2 + 6n + 1$ E) $n((2n+1)^2 - n^2)$

19. The numbers x, y, z and w satisfy $x+2y = 3z+w$ and $z+2w = 3x+y$. Which of the following statements is not necessarily correct?

A) $y + w = 2(z + x)$ B) $x - 3z = w - 2y$ C) $4x + 3y = 4z + 3w$
D) $z + 2w = 3(3z + w - 2y) + y$ E) $z + 3x = y + 2w$

20. The small circles have radius 1, the large one has radius 3. The circles are tangent to one another and the two parallel lines as shown. What is the distance between the touching points P and Q?



A) $2\sqrt{12}$ B) 8 C) $2\sqrt{20}$ D) 10 E) $2\sqrt{21}$