THE GEORG MOHR CONTEST 2011

First round

16 November 2010

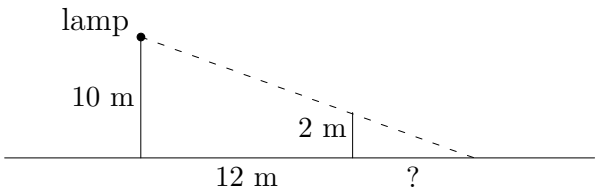
*Duration: 60 minutes*
*Aids permitted: none*
*Answer by ticking the enclosed answering sheet.*

1. "Think of a number. Add 3. Multiply the result by 4. Subtract 6 from the result. Divide the result by 2. Subtract 1." So Søren tells me, and if I now tell him which result I have got to he can tell me immediately which number I thought of originally. What can Søren do to my final result in order to get to my secret starting number?

   A) multiply it by 2  B) divide it by 2 and subtract 1
   C) multiply it by 2 and add 1  D) divide it by 4
   E) add 3

2. A 2 meter high pole throws a shadow in the light of a bright lamp on top of a 10 meter high post standing 12 meter from the pole. How long is the shadow?

   not drawn to scale




   A) 2.4 meter  B) 2.5 meter  C) 3 meter  D) 4.8 meter  E) 5 meter

3. Which of the following numbers is largest?

   A) $\frac{2100}{3000}$  B) $\frac{20}{31}$  C) $\frac{2000}{3011}$  D) $\frac{2}{3}$  E) $\frac{2100}{3011}$

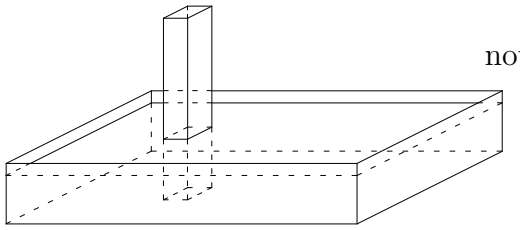
4. On five cards a symbol is printed on each side. One of the cards has the symbol ♣ on one side and ♠ on the other side, one carries the symbols ♠ and ♡, one has ♢ and ♣, one has ♡ and ♢, and, finally, there is one with ♢ and ♠. I shuffle the cards and lay them out in a row on the table. You see the symbols:

   ♢ ♣ ♠ ♣ ♢

   Which symbol is on the back of the card in the middle?

   A) ♢  B) ♡  C) ♠  D) ♣  E) it cannot be decided

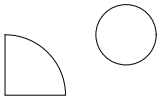
5. In a small ornamental pool, which is 150 cm on one side and 160 cm on the other side, stands a 120 cm high marble block whose base measures 20 cm × 30 cm. Normally, the water stands 40 cm high in the pool. One day the marble block topples into the water. How much does the water rise?



not drawn to scale

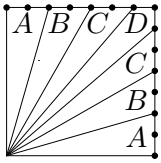
A) 1 cm B) 2 cm C) 3 cm D) 4 cm E) 5 cm

6. A circle with radius $r$ has the same area as a quarter circle with radius 1. What is $r$?



A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{\pi}{2}$ D) $\frac{\sqrt{\pi}}{2}$ E) $\frac{1}{4}$

7. A square chocolate layer cake measuring 35 cm × 35 cm is cut into seven neat pieces as shown. There is 5 cm between each of the marks shown. Which type of piece is smallest?




A) type $A$ B) type $B$ C) type $C$ D) type $D$
E) all of them have the same size

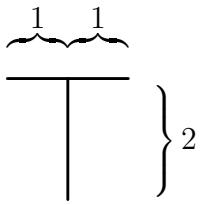
8. Three numbers $x$, $y$ and $z$ satisfy $xy + z = 0$. What can be deduced from this?

A) all the numbers $x$, $y$ and $z$ are 0 B) $x^2y^2 + z^2 = 0$
C) at least one of the numbers $x$, $y$ and $z$ is negative
D) if the number $z$ is 0, at least one of the numbers $x$ and $y$ is 0, as well
E) if the number $x$ is negative, at least one of the numbers $y$ and $z$ is positive

9. Nine small cardboard pieces are numbered with the numbers $1, 2, 3, \ldots, 9$ and put in a bag. The bag is shaken, and Marie sticks her hand into the bag and takes out two cardboard pieces. What is the probability of both the sum and the product of the numbers pulled out being odd?

A) 0 B) $\frac{1}{4}$ C) $\frac{5}{9}$ D) $\frac{25}{81}$ E) 1

10. What is the radius of the smallest circle that can cover the T shown?

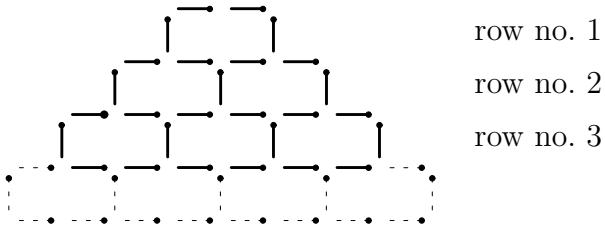



A) 1 B) $\sqrt{2}$ C) $\frac{5}{4}$ D) $\sqrt{3}$ E) $\sqrt{5}$

11. Which of the following numbers does not end in 5?

A) $5+15+25+35+45$ B) $5\cdot 15\cdot 25\cdot 35\cdot 45$
C) $19^2-16^2$ D) $\frac{20\cdot 35\cdot 9}{4\cdot 15}$ E) $5^{25}-25^5$

12. Adam is building a figure with many rows. He just finished row no. $n$. How many matches does he need to add to the figure so that it has $n+1$ rows?




A) $n+1$ B) $n+(n+1)+2+(n+2)$ C) $3(n+2)$
D) $2n+2(n+1)$ E) $2n$

13. The equation

$$\frac{x}{3}+\frac{5}{x}=45x+x^2$$

has three solutions. One of the solutions is:

A) $x=3$ B) $x=\frac{1}{3}$ C) $x=5$ D) $x=\frac{1}{5}$ E) $x=\frac{1}{15}$

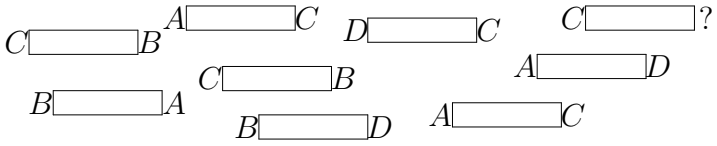
14. $A$ and $B$ play a game of dice with the following rules: $A$ throws a die. If the die shows 1 or 6 then $A$ has won. If it shows 2, 3 or 4 then $B$ has won. If it shows 5 then $A$ must throw it again, but this time 1 or 6 means that $B$ has won whereas 2, 3, 4 or 5 means that $A$ has won. What is the probability of $A$ winning this game?

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{4}{9}$ E) $\frac{5}{6}$

15. Anna needs to finish two subjects in order to have completed her education. If she gets 12 in both of them the average of her marks will be exactly 8. If she gets 2 in both subjects it will be exactly 7. Of how many subjects does the education consist?

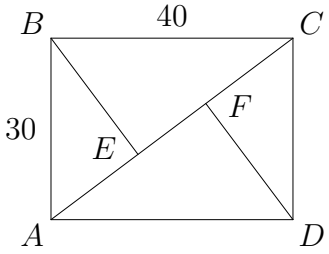
A) 4 B) 5 C) 10 D) 12 E) 20

16. Peter's railroad cars have a hinge at each end. Hinges of the type $A$ fit hinges of the type $B$, and hinges of the type $C$ fit hinges of the type $D$. With hinges that fit, the railroad cars can be joined. Unfortunately, the hinge shown with a question mark is broken. Which hinge must be put on in order that the cars shown can be assembled to one long train?




A) $A$ B) $B$ C) $C$ D) $D$
E) it is impossible whatever hinge is put on

17. In the rectangle $ABCD$ the sides are $AB = 30$ and $BC = 40$. The segments $BE$ and $DF$ are perpendicular to $AC$. How long is the segment $EF$?




A) 12 B) 14 C) 16 D) 18 E) 20

18. At most how many points can one place on a flat piece of paper so that by drawing at most three lines one can obtain that one cannot get from any point to any other point without crossing one of the lines?

A) 4 B) 6 C) 7 D) 8 E) 9

19. In the little snug hotel Syvkanten there are just enough rooms so that all bookings for the week of the fall vacation can be accommodated without any guests having to change their rooms during the stay. The individual bookings, indicated here by the arrival day of the week followed by the number of nights, are the following: (Mo, 3), (Fr, 2), (We, 3), (Mo, 4), (Th, 2), (Th, 3), (Mo, 2), (Tu, 1), (Tu, 4). How many rooms are there in the hotel?

A) 9 B) 6 C) 5 D) 7 E) 4

20. A three digit number $ABC$ is composed of three different digits $A$, $B$ and $C$, all of them odd. It is given that the sum of the numbers $ABC$, $BCA$ and $CAB$ has three digits. What is this sum?

A) 300 B) 683 C) 777 D) 999 E) it cannot be decided