THE GEORG MOHR CONTEST 2010

First round

24 November 2009

Duration: 60 minutes
Aids permitted: none
Answer by ticking the enclosed answering sheet

1. Which of the following numbers is greatest?

   A) $\frac{1}{4}$ B) 0.47 C) $\frac{4}{7}$ D) $(\frac{1}{4})^2$ E) $\frac{7}{40}$

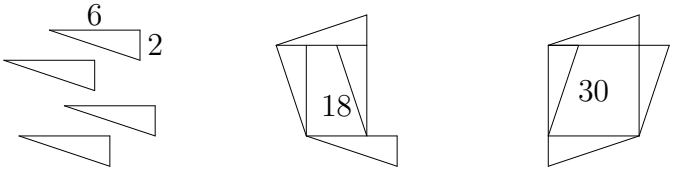
2. Bo is the second tallest boy in the class, but he is not as tall as Ane, who is the third tallest of the girls. Hence it follows that the number of pupils who are taller than Bo is

   A) 1 B) 2 C) 3 D) at most 4 E) at least 4

3. If $\frac{1}{n+3} = \frac{1}{7}$ then $\frac{1}{n^2+9}$ is equal to

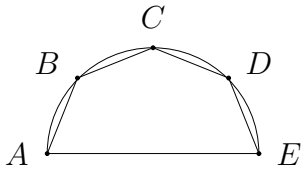
   A) $\frac{2}{7}$ B) $\frac{1}{14}$ C) $\frac{1}{49}$ D) $\frac{1}{16}$ E) $\frac{1}{25}$

4. What is the largest possible area that can be obtained for a quadrilateral domain bounded by the four pieces shown on the left? The figure shows a couple of examples. The pieces may be moved around and turned over as you like.



   A) 32 B) 36 C) 40 D) 42 E) 48

5. The points A, B, C, D and E lie at equal mutual distances on a semicircle as shown. How large is angle E in the pentagon $ABCDE$?



   A) 60° B) 45° C) 135° D) 72° E) 67,5°

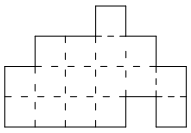
6. In which of the following equations is $x = 5$ a solution?

A) $(x-1)+(x-6)=0$ B) $(x-4)^2+(x-5)^2=0$
C) $(x-3)^3+(x-7)^3=0$ D) $(x-5)^4+(x-2)^4=0$
E) $(x-1)^5+(x-5)^5=0$

7. Anna threw a die three times and got three different numbers of pips. The sum of these numbers of pips is 11. In the first and third throws the die showed even numbers. None of the throws was a two. What did the die show in the second throw?

A) 1 B) 3 C) 4 D) 5 E) 6

8. One or more of the boring white tiles in the shown area are to be replaced, each of them, with a yellow or a black tile. How many possibilities are there for the new appearance of this tiled area?



A) $16 \cdot 15$ B) $3^{16}-1$ C) $2^{16}-1$ D) 16^2-1 E) 31

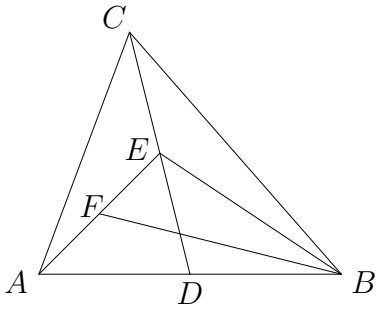
9. For a party, each of n persons prepares s liters of soup. The soup is to be filled into big jars holding d liters each. At least how many jars are needed when each jar must only be half filled?

A) $\frac{2ns}{d}$ B) $\frac{\frac{1}{2}ds}{n}$ C) $\frac{2s}{dn}$ D) $\frac{nds}{2}$ E) $\frac{ns}{2d}$

10. A number where all digits are 2's or 7's is called a *lucky number*. How many lucky numbers between 1 and 10 000 are divisible by 9?

A) none B) 8 C) 12 D) 72 E) more than 100

11. In triangle ABC the point D is the midpoint of AB, E the midpoint of CD and F the midpoint of AE. If the area of triangle ABC is 1, what is the area of triangle BEF?

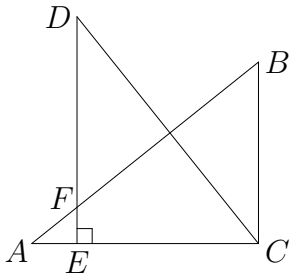


A) $\frac{1}{4}$ B) $\frac{1}{5}$ C) $\frac{1}{6}$ D) $\frac{1}{8}$ E) $\frac{2}{9}$

12. Choose by lot a random two-digit number (that is, a number among the numbers $10, 11, \ldots, 99$). Then exchange the digits. What is the probability of the resulting number being larger than the chosen one?

A) 50 % B) 40 % C) $\frac{4}{9}$ D) $\frac{36}{89}$ E) $\frac{7}{18}$

13. The triangles ABC and DCE are congruent (that is, similar and equally large), and $|AC| = 5$ and $|BC| = 4$ hold. What is the length of the segment DF?

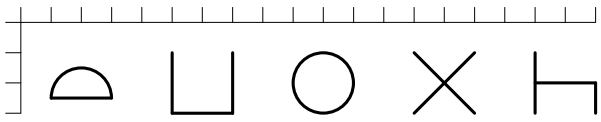


A) $3\frac{4}{5}$ B) 4 C) $4\frac{1}{5}$ D) $4\frac{1}{4}$ E) $4\frac{1}{3}$

14. A measuring device registers the results of a series of measurements and ends up displaying the tuple $[x; y; z]$, where x is the least measured value, y the number of measurements and z the average of the measured values. If $[3; 12; 5]$ is shown, what was then the highest measured value?

A) 12 B) 4 C) 7 D) 15 E) one cannot know

15. Each figure below is to be retraced with Indian ink. Which figure has the largest total length?



A) B) C) D) E)

16. At the course "Magic with food" each of the nine participants has learned one thing: no. 1 kan change ice cream into beer, no. 2 beer into ice cream, no. 3 flour into cabbage, no. 4 ice cream into cabbage, no. 5 cheese into beer, no. 6 cheese into flour, no. 7 flour into ice cream, no. 8 cabbage into cheese and no. 9 beer into flour. Between them, the participants can, in fact, change anything into anything. But if only eight of the participants have arrived, this is not necessarily possible. Which participant must not be absent?

A) no. 1 B) no. 4 C) no. 7 D) no. 8 E) no. 9

17. The price of each of five products, all of which in year 1990 cost 125 kroner, increases exponentially. A doubles its price in ten years, B's price is doubled in twelve years, C's price increases by 20 % every second year, D becomes 10 % more expensive annually and E cost 135 kroner in 1991. Which product is most expensive in 2010?

A) A B) B C) C D) D E) E

18. Peter has numbered 100 slips with the numbers from 1 to 100. He gives them out to his five children, equally many to each of them. Now every child must add its numbers and compare with the others. What is the largest possible difference that can occur between the results of two children?

A) 100 B) 20 C) 475 D) 1600 E) 500

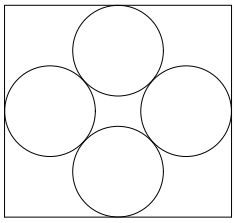
19. Set $a_0 = 3$ and $a_1 = 5$ and, thereafter,

$$a_2 = a_1 - a_0 \ , \quad a_3 = a_2 - a_1 \ , \quad \ldots$$

What happens to the numbers a_n in the long run?

A) the numbers get greater and greater
B) from a certain step all the numbers are positive
C) from a certain step all the numbers are negative
D) the numbers approach 0
E) the numbers repeat themselves periodically

20. Four circles of radius 1 are tangent to each other and the sides of a square as shown. What is the side length of the square?



A) 2 B) 4 C) $2+2\sqrt{2}$ D) $4+\frac{\sqrt{2}}{2}$ E) $2\sqrt{5}$