THE GEORG MOHR CONTEST 2009

First round

25 November 2008

Duration: 45 minutes
Tools allowed: none
Answer by ticking the answering sheet enclosed

1. At least how many different exam questions must a teacher prepare for a class of 22 students when each question may be used at most three times and when the last student must have a choice between at least four (not necessarily different) questions?
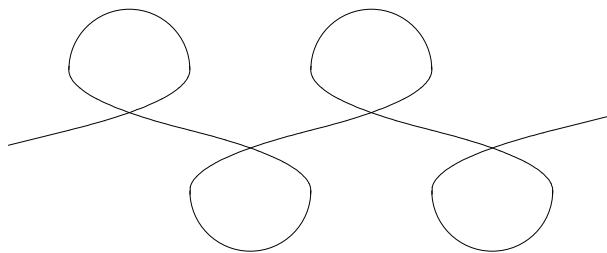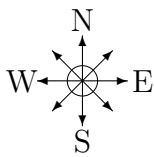
$$\text{A) } 7 \quad \text{B) } 8 \quad \text{C) } 9 \quad \text{D) } 10 \quad \text{E) } 12$$

2. At a party, n nice 8-person tables of the type shown are placed end to end. How many seats does one get?
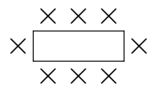
$$\text{A) } n \quad \text{B) } 7n+1 \quad \text{C) } n(n-2)+2 \quad \text{D) } 6n+2 \quad \text{E) } 8(n-1)$$

3. Peter and Marie are walking side by side with Peter to the right of Marie. They walk along the winding path in the park from the start in the west end to the exit towards the east. How many times on the way is Peter due north of Marie?
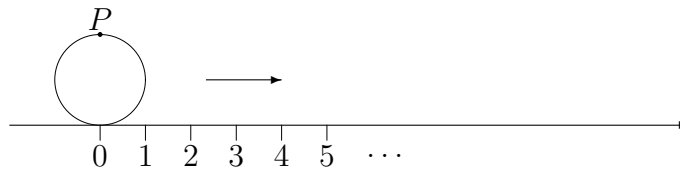

$$\text{A) } 0 \quad \text{B) } 1 \quad \text{C) } 2 \quad \text{D) } 4 \quad \text{E) } 5$$

4. Quite a few different numbers divide 20000. How many of these numbers lie between 4921 and 12138?

$$\text{A) } 2 \quad \text{B) } 3 \quad \text{C) } 4 \quad \text{D) } 10 \quad \text{E) } 54$$

5. A circle with radius 1 is rolled along a number line where all the positive integers 1, 2, 3, … are marked. Where does the point P hit one of these numbers for the first time? (The figure shows the starting position.)


A) at 2   B) at 22   C) at 157   D) at 392699   E) never

6. Ole lies on Mondays, Tuesdays and Wednesdays and tells the truth the rest of the week. Hans lies on Thursdays, Fridays and Saturdays and tells the truth the rest of the week.
"Yesterday I lied", Ole says one day. "So did I", Hans says.
On which day of the week were these statements made?

A)   Monday   B)   Wednesday   C)   Thursday
D)   Sunday   E)   it cannot be decided

7. All the children in the kindergarten are sitting in a circle on small coloured children's chairs. If everyone moves 4 places to the right, everyone will be sitting on a chair of the same colour as before. The same is true if everyone moves 9 places to the right. Hence we can infer that

A) if everyone moves 5 places to the left, nobody will be sitting on a chair of the same colour as before   B) there are at most 13 chairs
C) all chairs have the same colour   D) two neighbouring chairs always have different colours   E) there are at least 36 chairs

8. A coin with diameter $\frac{1}{2}$ is thrown at an infinite square grid consisting of squares of side length 1. What is the probability of the coin landing entirely within one of the squares?
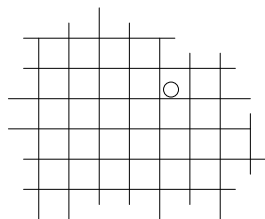

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) $\frac{\pi}{4}$ E) $\frac{4}{\pi^2}$

9. In a coordinate system, the points $P(212, 48)$, $Q(215, 52)$, $R(219, 50)$ and $S(222, 53)$ are given. Which line segment has the largest slope?

A) PQ B) PR C) PS D) QS E) RS

10. If m is an even number, and n is divisible by 6, which of the following numbers is then with certainty divisible by 4?

A) $m + n$ B) $mn - m$ C) $m^2 + n$ D) $m(m + n)$ E) $(m + 1)n$

11. How many real numbers x satisfy

$$(x-3)^2 + (2x-8)^2 + (2x-3)^4 + (x-4)^4 = 0 \ ?$$

A) 0   B) 1   C) 2   D) 3   E) 4

12. The three discs are spun independently. The player wins if one (or, as in the example shown, several) of them stops at M. What is the probability of winning?
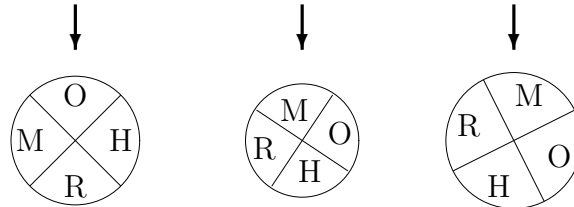

A) $\frac{1}{3}$ B) $\frac{1}{64}$ C) $\frac{3}{4}$ D) $\frac{27}{64}$ E) $\frac{37}{64}$

13. In Venneby you are friends with everybody in your family and with the friends of your friends. Those you are not friends with, you hate. We know that Anders and Bo are brothers of Carla, that Frede is friends with Tim, that Bo hates Dan and that Dan hates Tim's and Carla's daughter Else. Hence it follows that

A) Carla is friends with Frede   B) Else hates Frede
C) Anders hates Tim   D) Dan is friends with Tim
E) Frede hates Anders

14. The bumper cars nos. 1, 2, 3 and 4 are at a given moment at the positions shown. The arrows show the direction in which the cars are running, and the length of the arrows indicates the speed of the cars. Which cars will collide if they all continue with the same direction and speed?
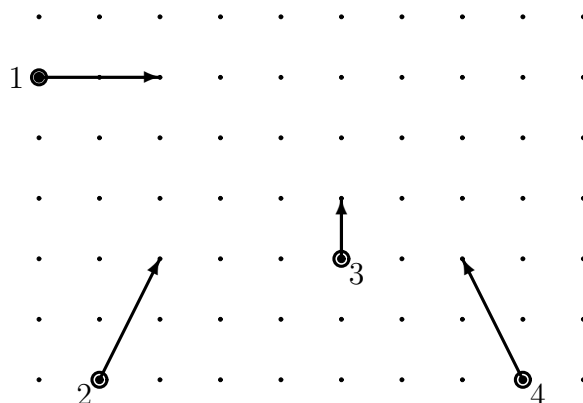
A) 1 and 2   B) 3 and 4   C) 1 and 3   D) 2 and 4
E) no collisions will take place

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15. Jan has agreed on the following code words with a friend: g = mouse, m = always, gm = hello, gg = fine, mg = blue, mm = Easter. One day he receives the message "ggmmmg". Unfortunately the spaces are lost, so maybe it says "fine Easter always mouse", maybe "mouse hello always blue", maybe something else. How many possible interpretations are there?

A) 9   B) 11   C) 13   D) 23   E) 36

16. The 10 students of a mathematics class tell you that they have got the following marks: 12, 4, 7, 7, −3, 10, 12, 02, 10, 7. But something is wrong, for the teacher can inform you that the average score of the team is only 5.5! What is the smallest possible number of students who have lied to you about their mark?

A) 1   B) 2   C) 3   D) 4   E) 5

17. The numbers a and b both lie between 0 and 1. Which of the following numbers does not necessarily have this property?

A) $\frac{a+b}{2}$ B) $\sqrt{ab}$ C) $\sqrt{\frac{a}{2}} + \sqrt{\frac{b}{2}}$ D) $\frac{ab}{a+b}$ E) a^2b^2
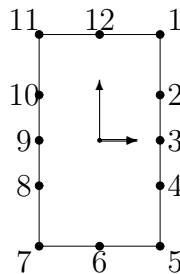
18. Which of the following numbers is not a perfect square?

A) 57121   B) 313600   C) 34969   D) 392025   E) 119716

19. Let $a_0 = 2$, and let $a_n = \frac{1}{2}(a_{n-1} + \frac{2}{a_{n-1}})$ for $n = 1, 2, 3, \dots$.
For large values of n, the number a_n will

A) approach 1   B) approach 2   C) approach $\frac{\pi}{2}$ D) approach $\sqrt{2}$
E) oscillate between very large and very small values

20. On a classic clockwork, an elegant face is mounted. It is formed as a rectangle where 1 o'clock is in the upper right corner (see sketch). The distance between the markings of 12 o'clock and 1 o'clock measured along the edge of the rectangle is 1. How long is the distance between the markings of 1 o'clock and 2 o'clock?

A) $\frac{2\sqrt{3}}{3}$ B) $\frac{1}{\sqrt{3}}$ C) $\frac{\sqrt{3}}{2}$ D) $1 + \frac{\sqrt{3}}{2}$ E) $2 - \frac{\sqrt{3}}{2}$

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