THE GEORG MOHR CONTEST 2007

First round

October 31, 2006

Duration: 45 minutes
Tools allowed: none
Answer by ticking the answering sheet enclosed

1. At IceIceIce, a cone with two balls of homemade ice cream costs 15 kroner, a cone with three balls 20 kroner, one with four balls 25 kroner etc. How much does a cone with n balls cost?

   A) $15n + 5$ kr. B) $5n + 10$ kr. C) $5n + 5$ kr. D) $n + 15$ kr. E) $(n - 2) \cdot 5 + 10$ kr.

2. These are four words written in a secret code:

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   You are told that the four words mean BAS, OLE, ABE and SKO. You do not know, however, which word means what. What is written here?

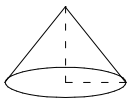
   + ! [

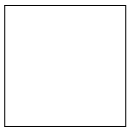
   A) SOL B) BOA C) LOK D) BAL E) LAS

3. Among the 20 pupils in Marie's class, 13 pupils have visited Spain, and 12 pupils have visited Italy. Four of the pupils have visited neither Spain nor Italy. How many pupils have visited both Spain and Italy?

   A) 8 B) 9 C) 10 D) 12 E) 13

4. A cone with radius 60 cm og height 80 cm must pass through a quadratic window. What is the side length of the smallest window that allows this among those mentioned?





   A) 80 cm B) 100 cm C) 120 cm D) 140 cm E) 160 cm

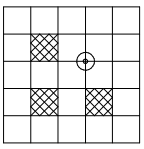
5. Anton is younger than Bo, Dan is older than Esben, Carl is younger than Dan, Esben is younger than both Carl and Bo, and Bo is older than Carl. Which of the boys is oldest?

A) Anton B) Bo C) Carl D) Dan E) cannot be answered

6. The girls in Marie's class are at the movies. Their seats have the numbers 1, 3, 5, 7, etc. The sum of all these numbers is 144. How many girls are there?

A) 13 B) 10 C) 12 D) 9 E) 11

7. The floor of the room in the turret is covered with quadratic tiles except at the three places shown, where the tile is replaced by a column that reaches the ceiling. How large an area of the floor, measured in number of tiles, is illuminated by the lamp? (The position of the lamp is shown by a circle.)



A) $15\frac{1}{2}$ B) 13 C) $14\frac{1}{2}$ D) $14\frac{2}{3}$ E) 15

8. In how many ways may a group of six persons be divided into two groups of three persons?

A) 9 B) 10 C) 18 D) 20 E) 36

9. On Særø, two sorts of coin are used: seven kroner coins, whose weight is 3 grams, and five kroner coins, whose weight is 2 grams. What is the least possible weight of the amount 127 kroner?

A) 18 g B) 19 g C) 25 g D) 51 g E) 54 g

10. Which of the following problems cannot be solved? Write a 5-digit number composed of all the digits 1, 2, 3, 4 and 5 so that the number is divisible by

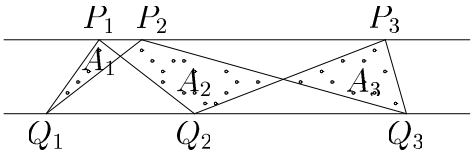
A) 4 B) 5 C) 6 D) 9 E) 125

11. What is the number of solutions to the equation

$$x - x(x-1) + x(x-2) - x(x-1)(x-2) = 0 \ ?$$

A) 0 B) 1 C) 2 D) 3 E) 4

12. The points P_1, P_2, P_3 and Q_1, Q_2, Q_3 lie on two parallel lines, and they are connected by line segments as shown. What is true about the areas A_1, A_2 and A_3?

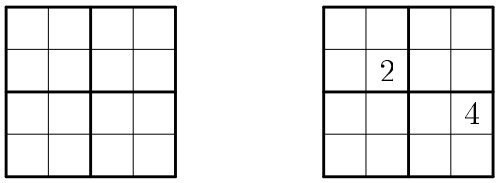


A) $A_1 + A_3 = A_2$ B) $A_1 + A_3 < A_2$ C) $\sqrt{A_1} + \sqrt{A_3} = \sqrt{A_2}$
D) $\frac{1}{3}A_1 + \frac{1}{3}A_3 = \frac{1}{2}A_2$ E) none of these

13. As a password to a discussion group on data security, a person used the nine lettes in the word GEORGMOHR, however in a different order – which he forgot, unfortunately. He remembers only that M was the first letter, that H was the last one, and that identical letters followed each other immediately. How many possibilities are there?

A) 24 B) 48 C) 72 D) 105 E) 720

14. Peter is constructing mini sudoku puzzles for his younger brothers and sisters. He writes some numbers in advance. Then the puzzle consists in filling in the rest of the squares so that each row and each column and each 2×2-box contains the numbers 1–4. In the example on the right, the puzzle may be solved in many different ways. What is the largest number of squares filled in in advance which one can have in a mini sudoku which has more than one solution?



A) 5 B) 8 C) 10 D) 12 E) 13

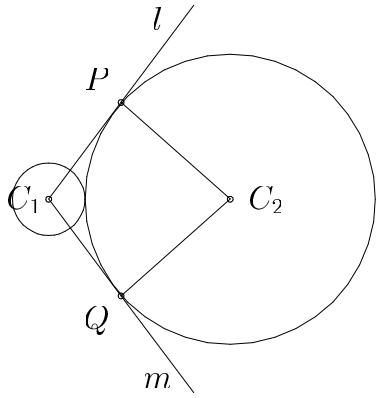
15. Which number is greatest?

A) $2^3 \cdot 3^4 \cdot 4^3$ B) $(4 \cdot 3^2)^2 \cdot (3 \cdot 4^2) \cdot 2$
C) $2 \cdot 3 \cdot \frac{4}{2 \cdot 3} \cdot 3 \cdot \frac{3^3}{2^4} \cdot 2^{10}$ D) $2^{10} \cdot 3^4$ E) $\frac{3^4}{2} \cdot 4^5$

16. Which of the following equations *doesn't* have the number $\frac{\sqrt{2}}{2}$ as a solution?

A) $x^2 = \frac{1}{2}$ B) $2x = \sqrt{2}$ C) $\sqrt{2}\, x = 1$
D) $\sqrt{1 - 2x^2} = 0$ E) $\frac{1}{2}x^4 = \frac{1}{8}x^2$

17. The small circle has centre C_1 and radius 3, and the large circle has centre C_2 and radius 12. The lines l and m pass through C_1 and touch the large circle at the points P and Q. What is the area of quadrilateral C_1PC_2Q?



A) 81 B) 108 C) 144 D) 162 E) 225

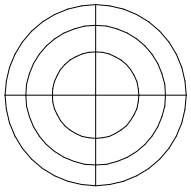
18. Someone has calculated n^{16} for all values of n from 1 to 100 and carefully written the last digit of each result on a piece of paper. How many different digits appear on this piece of paper?

A) 2 B) 4 C) 6 D) 8 E) 10

19. A, B and C are playing the following game. Each of them writes one of the numbers 1, 2 or 3 on a slip which the other players do not see. The player whose number is closest to the average of all three numbers wins the prize. If more than one player gets closest, the prize is divided equally among those getting closest. C wants to maximize his expected winnings. He knows that A and B choose their numbers completely randomly. What would be most profitable for C to write?

A) 1 B) 2 C) 3 D) 1 and 3 are best
E) 1, 2 and 3 are equally good

20. All 12 sections have the same area. The circle situated between two other circles has radius 1. What is the radius of the outmost circle?



A) $\frac{2}{\sqrt{3}}$ B) $\sqrt{2}$ C) π D) $\sqrt{\frac{3}{2}}$ E) $\frac{\sqrt{3}}{2}$