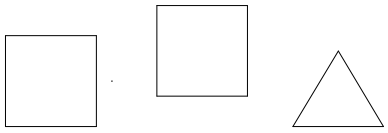
THE GEORG MOHR CONTEST 2006

First round

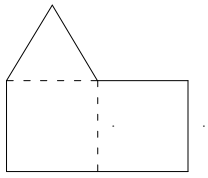
November 1, 2005

Duration: 45 minutes
Tools allowed: none
Answer by ticking the answering sheet enclosed

1. Two squares and an equilateral triangle, all of them with the same side length, are to be assembled to a single figure. How many different figures can be made? (Figures are considered identical if they can cover each other or are mirror images of each other.)

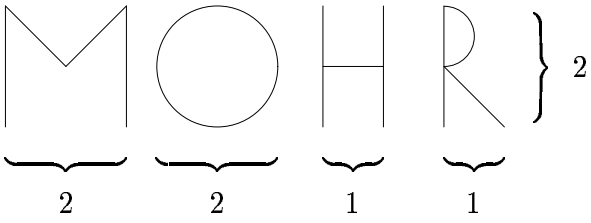


Example:



A) 3 B) 6 C) 12 D) 24 E) 5

2. The *diameter* of a figure is understood to be the diameter of the smallest possible circle that can cover the figure. Which of the following figures has the largest diameter?



A) M B) O C) H D) R
E) two of the figures have the largest diameter

3. Which of the following numbers is not divisible by 3?

A) 333+45519 B) 12445+54422 C) 2777+7772 D) 9416+7864
E) 212121+414141

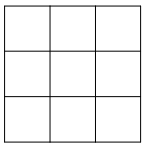
4. How many children are there in the family? We ask Georg. "I have three brothers," Georg tells us, and he adds with a smile: "And each of my brothers has three sisters. Now I think you can figure out the answer!"

A) 7 B) 8 C) 9 D) 10 E) 12

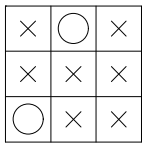
5. Which fraction is largest?

   A) $\frac{276-135}{423-314}$ B) $\frac{314-135}{423-276}$ C) $\frac{423-314}{276-135}$ D) $\frac{314-276}{423-135}$ E) $\frac{423-135}{314-276}$

6. In how many different ways can the diagram be filled in with crosses and circles?



Example:

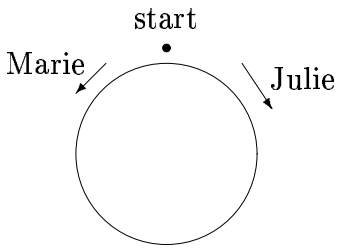


   A) $9 \cdot 8 \cdot \ldots \cdot 2 \cdot 1$ B) $2 \cdot 9$ C) 2^9 D) 9^2 E) $3 \cdot 2 \cdot 2$

7. If all cute animals are furry, all green animals are cute, all small animals are cute, and all cute, furry animals are green, which of the following statements is then not necessarily correct?

   A) all small animals are furry B) all cute animals are green
   C) all furry animals are green D) all small animals are green
   E) all small, green animals are furry

8. Marie and Julie run round the big, round flower bed in different directions. When Julie has got round the bed twice, and Julie three times, they meet each other again at the starting point for the first time. How many times did they pass each other on the way?



   A) 2 B) 3 C) 4 D) 6 E) 8

9. The number

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}$$

   is equal to

   A) $\frac{25}{30}$ B) $\frac{464}{121}$ C) $\frac{463}{120}$ D) $\frac{464}{123}$ E) $\frac{4819}{3840}$

10. Eight small ordinary dice are glued together to one larger cube. What is the probability that none of the small dice will show the face 'one'?

    A) $\frac{1}{8}$ B) $\frac{1}{3}$ C) $\frac{1}{256}$ D) $\frac{1}{8} \cdot \frac{1}{7} \cdot \frac{1}{6} \cdot \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2}$ E) $\frac{1}{8} \cdot \frac{3}{6}$

11. The numbers a, b, c and d satisfy the equations

$$a+b-c=d \ \wedge \ a-b+c=d \ \wedge \ -a+b-c=d \ \wedge \ -a-b+c=d \ .$$

What can be deduced from this? (The symbol "$\wedge$" means "and".)

A) $a=b \ \wedge \ d=0$ B) $b=c \ \wedge \ a=0$ C) $a=c \ \wedge \ d=0$
D) $b=d \ \wedge \ a=0$ E) $a=d \ \wedge \ c=0$

12. Peter is going to meet some of his friends. He does not know exactly how many they will be altogether; every number between two and eight is possible. On his way he passes a huge bowl of sweets, and he is allowed to take as many of them as he wants. What is the smallest number of sweets Peter must take to be sure that the sweets can be divided among him and his friends in such a way that everyone gets the same number?

A) 960 B) 1680 C) 420 D) 840 E) 40320

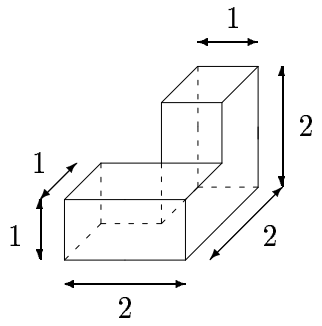
13. The number

$$\frac{5-2}{\sqrt{5}-\sqrt{2}}$$

is equal to

A) $\frac{21}{3}$ B) $\sqrt{3}$ C) $\sqrt{5}-\sqrt{2}$ D) $5\sqrt{2}-2\sqrt{5}$ E) $\sqrt{5}+\sqrt{2}$

14. How many blocks of the type shown are required to build a cube with side length 4?



A) 4 B) 6 C) 8 D) 16 E) it is not possible

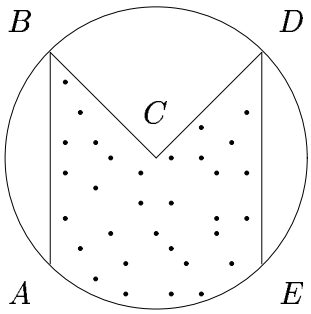
15. A cylinder with diameter $2r$ and height $2r$ has the same total surface area as a cube. What is the area of each face of the cube?

A) $4r^2$ B) πr^2 C) $r\pi^2$ D) $\frac{4}{3}\pi r$ E) $\frac{1}{3}\pi r^2$

16. On a blackboard are first written the numbers $a_1 = 2$ and $a_2 = 8$. Then numbers $a_3, a_4, ...$ are written according to the following system: Each written number is the average of *all* the numbers that are already present on the blackboard. What is the number a_{10}?

A) 3 B) 4,666... C) 4 D) 4,5 E) 5

17. The letter M is inscribed in a circle with centre C and radius 1 as shown. The segments AB and ED are parallel, and the angles at B and D are 45°. What is the area of the dotted domain?



A) $1+\frac{\pi}{4}$ B) $1+\frac{\pi}{3}$ C) $\pi^2-\frac{1}{3}$ D) $\frac{1}{2}+\frac{2\pi}{3}$ E) $1+\frac{\pi^2}{6}$

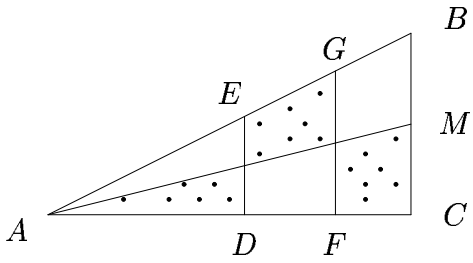
18. The numbers x, y and z are positive integers larger than 1 obeying $xyz = 190$. What can be deduced from this?

A) x is an even number B) $y < \sqrt{190}$ C) $x+y+z = 26$
D) $x+y+z = 28$ E) $x+y+z > 29$

19. Let $n \geq 2$ be an integer. What is the condition for the existence of a $2n$-gon where n sides are mutually parallel and the remaining n sides are also mutually parallel?

A) that n is even B) that n is odd C) that $n = 2$
D) it is not possible for any $n \geq 2$ E) it is possible for all $n \geq 2$

20. Triangle ABC, where AC is horisontal and BC vertical, is divided by two vertical segments DE and FG. We are informed that the three dotted domains have the same area, that $BM = MC = 1$, and that $AC = 3\sqrt{3}$. What is the length of segment DF?



A) $3\sqrt{2}-3$ B) $3\sqrt{2}+1$ C) $3-\frac{3\sqrt{3}}{2}$ D) $\sqrt{3}$ E) $3\sqrt{3}-2$