The Georg Mohr Contest 2018
Second Round

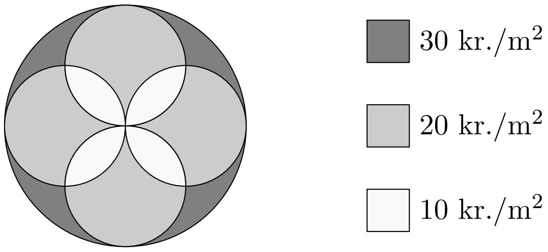
Tuesday, January 9th, 2018 at 9–13

Aids permitted: only writing and drawing tools.
Remember that your arguments are important in the assessment
and that points may also be awarded to partial answers.

Problem 1. A blackboard contains 2018 instances of the digit 1 separated by spaces. Georg and his mother play a game where they take turns filling in one of the spaces between the digits with either a $+$ or a $\times$. Georg begins, and the game ends when all spaces have been filled. Georg wins if the value of the expression is even, and his mother wins if it is odd.

Which player may prepare a strategy which secures him/her victory?

Problem 2. The figure shows a large circle with radius 2 m and four small circles with radii 1 m. It is to be painted using the three shown colours.



What is the cost of painting the figure?

Problem 3. The positive integers a, b and c satisfy that the three fractions

$$\frac{b}{a}, \quad \frac{c+100}{b} \quad \text{and} \quad \frac{a+b+169}{2c+200}$$

are all integers.

Determine all possible values of a.

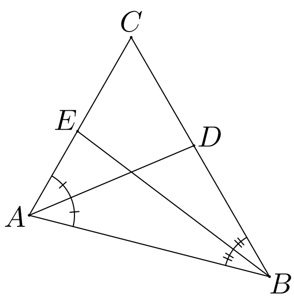
Problem 4. A sequence $a_1, a_2, a_3, \ldots, a_{100}$ of 100 (not necessarily distinct) positive numbers satisfy that the 99 fractions

$$\frac{a_1}{a_2}, \frac{a_2}{a_3}, \frac{a_3}{a_4}, \ldots, \frac{a_{99}}{a_{100}}$$

are all distinct.

How many distinct numbers must there be, at least, in the sequence $a_1, a_2, a_3, \ldots, a_{100}$?

Problem 5. In triangle ABC the angular bisector from A intersects the side BC at the point D, and the angular bisector from B intersects the side AC at the point E. Furthermore $|AE| + |BD| = |AB|$.



Prove that $\angle C = 60°$.

Sponsors: Undervisningsministeriet, Jobindex, Georg Mohr Fonden and Matematiklærerforeningen.