## The Georg Mohr Contest 2015 Second Round

Tuesday, January 6th, 2015 at 9–13

Aids permitted: only writing and drawing tools. Remember that your arguments are important in the assessment, and that points may also be awarded to partial answers.

**Problem 1.** The numbers a, b, c, d og e satisfy

$$a + b < c + d < e + a < b + c < d + e$$
.

Which of the numbers is the smallest, and which is the largest?

**Problem 2.** The numbers  $1, 2, 3, \ldots, 624$  are paired in such a way that the sum of the two numbers in each pair is 625. For example 1 and 624 form a pair, and 30 and 595 form a pair.

In how many of the 312 pairs does the smaller number evenly divide the larger?

## Problem 3.

Triangle ABC is equilateral. The point D lies on the continuation of AB beyond B, the point E lies on the continuation of CB beyond B, and |CD| = |DE|.

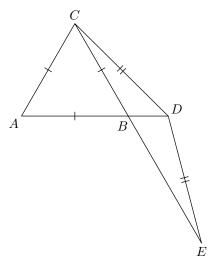
Prove that |AD| = |BE|.

**Problem 4.** Determine all numbers x, y and z satisfying the system of equations

$$x^{2} + yz = 1$$
  

$$y^{2} - xz = 0$$
  

$$z^{2} + xy = 1.$$



**Problem 5.** For which numbers n is it possible to put marks on a stick such that all distances 1 cm, 2 cm, ..., n cm each appear exactly once as the distance between two of the marks, and no other distance appears as such a distance?

Sponsors: Undervisningsministeriet, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Georg Mohr Fonden, Matematiklærerforeningen, Dansk Matematisk Forening, Gyldendal and Aarhus Universitetsforlag.