The Georg Mohr Contest 2010
Second round

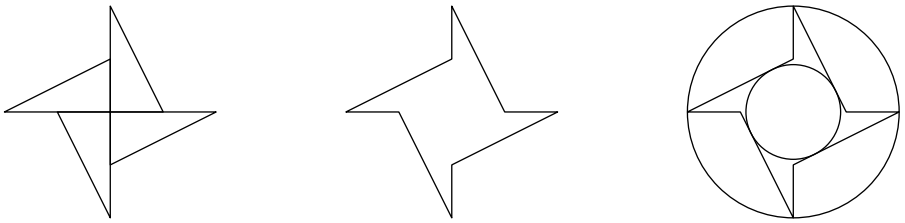
Thursday, January 7, 9:00 – 13:00

Aids permitted: Only writing and drawing tools

Remember to justify all of your answers

Problem 1. Four right triangles, each with the sides 1 and 2, are assembled to a figure as shown.

How large a fraction does the area of the small circle make up of that of the big one?

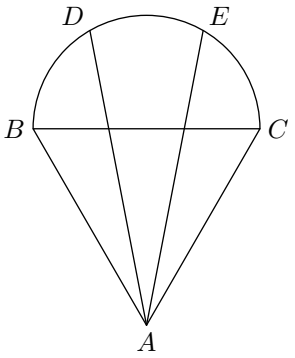


Problem 2. Prove that for every integer n there exist integers a, b, c such that $n = a^2 + b^2 - c^2$.

Problem 3. Can 29 boys and 31 girls be lined up in a row holding hands such that no one is holding hands with two girls?

Problem 4. An equilateral triangle ABC is given. With BC as diameter, a semicircle is drawn outside the triangle. On the semicircle, points D and E are chosen such that the arc lengths BD, DE and EC are equal.

Prove that the line segments AD and AE divide the side BC into three equal parts.



Problem 5. It is given that 2^{2010} is a 606-digit number beginning with 1.

How many of the numbers $1, 2, 2^2, 2^3, \ldots, 2^{2009}$ begin with 4?

Sponsors: Georg Mohr Fonden, Carlsbergs Mindelegat for Brygger J.C. Jacobsen, Dansk Matematisk Forening, Matematiklærerforeningen, Undervisningsministeriet, Gyldendal, Syddansk Universitetsforlag and Texas Instruments.