The Georg Mohr Contest 2008
Second round

Thursday 17 January 2008 at 9–13 hours

Tools for writing and drawing are the only ones allowed

Problem 1.   Denmark has played a football match against Georgia. The match ended 5-5, and between the first and the last goal, the score was at no time even. No country scored three successive goals, and Denmark scored the sixth goal.
Can one determine from this information which country scored the fifth goal?

Problem 2.   It is true of three integers p, q and r that $p + q^2 = r^2$.
Prove that 6 divides pqr.

Problem 3.   The numbers from 1 to 500 are written on the blackboard. Two players A and B alternately erase a number, and A erases the first number. If the sum of the last two numbers on the blackboard is divisible by 3, B wins, otherwise A wins.
Which player can choose a strategy that secures the victory of this player?

Problem 4.   In triangle ABC we have $AB = 2$, $AC = 6$ and $\angle A = 120°$. The bisector of angle A intersects the side BC at the point D.
Determine the length of AD. The answer must be given as a fraction with integer numerator and denominator.

Problem 5.   For every positive integer n, the numbers 2^n and 5^n form a number t_n consisting of the digits of 2^n followed by the digits of 5^n. E.g. $t_4 = 16625$.
How many digits has the number t_{2008}?