The Georg Mohr Contest in Mathematics 2003

Thursday 9 January 2003 at 9-13 h

Tools for writing and drawing are the only ones allowed

Problem 1.   In a right-angled triangle, the sum $a + b$ of the sides enclosing the right angle equals 24 while the length of the altitude h_c on the hypotenuse c is 7.
Determine the length of the hypotenuse.

Problem 2.   Solve the equation
$$x^5 + \lfloor x \rfloor = 20,$$
where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

Problem 3.   Determine the integers n that make

$$\mid 2n^2 + 9n + 4 \mid$$

a prime number.

Problem 4.   Georg and his mother love pizza. They buy a pizza shaped as an equilateral triangle. Georg demands to be allowed to divide the pizza by a straight cut and then make the first choice. The mother accepts this reluctantly, but she wants to choose a point of the pizza through which the cut must pass.
Determine the largest fraction of the pizza which the mother is certain to get by this procedure.

Problem 5.   For what integers $n \geq 2$ may the numbers from 1 to 16 be arranged in a quadratic scheme such that the four sums of the elements in a row and the four sums of the elements in a coloumn are all mutually different and divisible by n?